The Fiscal Arithmetic of a Slowdown in Trend Growth Online Appendix

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Baseline Model Equations Α

Non-Stationary Equations A.1

$$\Lambda_t(1+\tau_{c,t}) = \frac{\zeta_t}{C_t - hC_{t-1}} - \beta h \mathbb{E}_t \left\{ \frac{\zeta_{t+1}}{C_{t+1} - hC_t} \right\}$$
(1)

$$\Lambda_t = \beta R_t \mathbb{E}_t \Lambda_{t+1}$$
(2)
$$= \mathbb{E}_t \left\{ (1 + \kappa^B P_{t+1}^B) \right\}$$

$$P_t^B = \frac{\Delta t \left[\left(1 + h + 1 + 1 \right) \right]}{R_t}$$

$$\Lambda_t = \beta R_t^F \mathbb{E}_t \Lambda_{t+1}$$
(3)
(4)

$$\beta R_t^r \mathbb{E}_t \Lambda_{t+1}$$

$$\Phi_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} (1 - \tau_{K,t+1}) r_{t+1}^K + \Phi_{t+1} (1 - \delta) \right]$$
(5)

$$\Lambda_t = \Phi_t \zeta_t^I \left[1 - \Upsilon \left(\frac{I_t}{I_{t-1}} \right) - \Upsilon' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \beta \mathbb{E}_t \Phi_{t+1} \zeta_{t+1}^I \Upsilon' \left(\frac{I_{t+1}}{I_t} \right) \frac{I_{t+1}^2}{I_t^2}$$
(6)

$$\gamma \zeta_t \zeta_t^L L_t^\nu = \Lambda_t (1 - \tau_{w,t}) W_t \tag{7}$$

$$K_{t} = (1 - \delta) K_{t-1} + \zeta_{t}^{I} \left[1 - \Upsilon \left(\frac{I_{t}}{I_{t-1}} \right) \right] I_{t}$$

$$(8)$$

$$V = K_{t}^{\alpha} - (Z, L)^{1-\alpha}$$

$$Y_t = K_{t-1}(Z_t L_t) \tag{9}$$

$$K \qquad Y_t \tag{10}$$

$$r_t = \alpha \frac{1}{K_{t-1}} \tag{10}$$

$$W_t = (1 - \alpha) \frac{1}{L_t} \tag{11}$$

$$NX_{t} = Y_{t} - C_{t} - I_{t} - G_{t}$$
(12)

$$CA = NX + (P^{F} - 1)P^{F}$$
(12)

$$CA_{t} = NX_{t} + (R_{t-1}^{t} - 1)B_{t-1}^{t}$$

$$B_{t}^{F} = B_{t-1}^{F}B_{t-1}^{F} + NX_{t}$$
(13)
(13)

$$B_t^F = R_t^* \exp\left[-\psi_b \left(\frac{b_t^F}{y_t} - \frac{b^F}{y}\right) + \zeta_t^b\right]$$
(14)

(15)

$$G_t = P_t^B B_t - (1 + \kappa^B P_t^B) B_{t-1} + \tau_{c,t} C_t + \tau_{w,t} W_t L_t + \tau_{K,t} r_t^K K_{t-1} + T R_t$$
(16)

$$\log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + \varepsilon_{z,t}$$
(17)

$$\log R_t^* = (1 - \rho_{R^*}) \log R^* + \rho_{R^*} \log R_{t-1}^* + \varepsilon_{R^*,t}$$
(18)

$$\log g_t = (1 - \rho_g^1 - \rho_g^2) \log g + \rho_g^1 \log g_{t-1} + \rho_g^2 \log g_{t-2} - (1 - \rho_g^1 - \rho_g^2) \gamma_{gb} (by_{t-1} - by) + \varepsilon_{g,t}$$
(19)
$$\tau_{c,t} = (1 - \rho_c^1 - \rho_c^2) \tau_c + \rho_c^1 \tau_{c,t-1} + \rho_c^2 \tau_{c,t-2} + (1 - \rho_c^1 - \rho_c^2) \gamma_{cb} (by_{t-1} - by) + \varepsilon_{c,t}$$
(20)

$$\tau_{w,t} = (1 - \rho_w^1 - \rho_w^2)\tau_w + \rho_w^1 \tau_{w,t-1} + \rho_w^2 \tau_{w,t-2} + (1 - \rho_w^1 - \rho_w^2)\gamma_{wb} (by_{t-1} - by) + \varepsilon_{w,t}$$
(21)

$$\tau_{K,t} = (1 - \rho_K^1 - \rho_K^2)\tau_K + \rho_K^1 \tau_{K,t-1} + \rho_K^2 \tau_{K,t-2} + (1 - \rho_K^1 - \rho_K^2)\gamma_{Kb} (by_{t-1} - by) + \varepsilon_{K,t}$$
(22)

$$\tau_{t} = (1 - \rho_{\tau} - \rho_{\tau})\tau + \rho_{\tau}\tau_{t-1} + +\rho_{\tau}\tau_{t-2} + (1 - \rho_{\tau} - \rho_{\tau})\gamma_{\tau b} (by_{t-1} - by) + \varepsilon_{\tau,t}$$
(23)
$$\log \zeta_{t} = \rho_{\varepsilon} \log \zeta_{t-1} + \varepsilon_{\varepsilon,t}$$
(24)

$$\log \zeta_t = \rho_t \log \zeta_{t-1} + \varepsilon_{\zeta,t} \tag{24}$$

$$\log \zeta_L = \rho_t \log \zeta_{t-1}^L + \varepsilon_{t-t} \tag{25}$$

$$\log \zeta_t^I = \rho_I \log \zeta_{t-1}^I + \varepsilon_{I,t}$$

$$(26)$$

$$\zeta_t^b = (1 - \rho_b)\zeta^b + \rho_b\zeta_{t-1}^b + \varepsilon_{b,t}$$
(27)

A.2 Stationary Equations

The normalised variables are as follows:

- 1. $c_t = \frac{C_t}{Z_t}$ 2. $\lambda_t = \Lambda_t Z_t$ 3. $\phi_t = \Phi_t Z_t$ 4. $i_t = \frac{I_t}{Z_t}$ 5. $k_t = \frac{K_t}{Z_t}$ 7. $r_t^F = R_t^F$ 8. $r_t = R_t$ 9. $w_t = \frac{W_t}{Z_t}$ 10. $y_t = \frac{Y_t}{Z_t}$ 11. $b_t^F = \frac{B_t^F}{Z_t}$ 13. $nx_t = \frac{NX_t}{Z_t}$ 15. $r_t^* = R_t^*$ 16. $g_t = \frac{G_t}{Z_t}$
- 5. $k_t = \frac{K_t}{Z_t}$ 6. $l_t = L_t$ 11. $b_t^F = \frac{B_t^F}{Z_t}$ 16. $g_t = \frac{G_t}{Z_t}$ 17. $\tau_t = \frac{TR_t}{T_t}$

6.
$$l_t = L_t$$
 12. $b_t = \frac{D_t}{Z_t}$ 17. $\tau_t = \frac{T R_t}{Z_t}$

$$\lambda_t (1 + \tau_{c,t}) = \frac{\zeta_t z_t}{c_t z_t - h c_{t-1}} - \beta h \mathbb{E}_t \left\{ \frac{\zeta_{t+1}}{c_{t+1} z_{t+1} - h c_t} \right\}$$
(28)

$$\lambda_t = \beta r_t \mathbb{E}_t \left(\frac{\tau_{t+1}}{z_{t+1}} \right)$$

$$p_t^B = \frac{\mathbb{E}_t \left\{ (1 + \kappa^B p_{t+1}^B) \right\}}{(30)}$$

$$\lambda_t = \beta r_t^F \mathbb{E}_t \left(\frac{\lambda_{t+1}}{z_{t+1}} \right)$$
(31)

$$\phi_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{z_{t+1}} (1 - \tau_{K,t+1}) r_{t+1}^K + \frac{\phi_{t+1}}{z_{t+1}} (1 - \delta) \right]$$
(32)

$$\lambda_t = \phi_t \zeta_t^I \left[1 - \Upsilon \left(\frac{i_t z_t}{i_{t-1}} \right) - \Upsilon' \left(\frac{i_t z_t}{i_{t-1}} \right) \frac{i_t z_t}{i_{t-1}} \right] + \beta \mathbb{E}_t \frac{\phi_{t+1} \zeta_{t+1}^I}{z_{t+1}} \Upsilon' \left(\frac{i_{t+1} z_{t+1}}{i_t} \right) \left(\frac{i_{t+1} z_{t+1}}{i_t} \right)^2$$
(33)

$$\gamma \zeta_t \zeta_t^L l_t^\nu = \lambda_t (1 - \tau_{w,t}) w_t \tag{34}$$

$$k_t = (1 - \delta) \frac{k_{t-1}}{z_t} + \zeta_t^I \left[1 - \Upsilon \left(\frac{i_t z_t}{i_{t-1}} \right) \right] i_t$$
(35)

$$y_t = k_{t-1}^{\alpha} l_t^{1-\alpha} z_t^{-\alpha} \tag{36}$$

$$r_t^K = \alpha \frac{g_t z_t}{k_{t-1}} \tag{37}$$

$$w_t = \frac{(1-\alpha)y_t}{l_t} \tag{38}$$

$$nx_t = y_t - c_t - i_t - g_t (39)$$

$$ca_{t} = nx_{t} + (r_{t-1}^{F} - 1)\frac{b_{t-1}^{F}}{z_{t}}$$

$$\tag{40}$$

$$b_t^F = \frac{r_{t-1}^F}{z_t} b_{t-1}^F + nx_t \tag{41}$$

$$r_t^F = r_t^* \exp\left[-\psi_b \left(\frac{b_t^F}{y_t} - \frac{b^F}{y}\right) + \zeta_t^b\right]$$
(42)

$$g_t + \frac{(1 + \kappa^B p_t^B)b_{t-1}}{z_t} = p_t^B b_t + \tau_{c,t} c_t + \tau_{w,t} w_t l_t + \frac{\tau_{K,t} r_t^K k_{t-1}}{z_t} + \tau_t$$
(43)

$$\log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + \varepsilon_{z,t}$$
(44)

$$\log r_t^* = (1 - \rho_{r^*}) \log r^* + \rho_{r^*} \log r_{t-1}^* + \varepsilon_{r^*,t}$$
(45)

$$\log g_t = (1 - \rho_g^1 - \rho_g^2) \log g + \rho_g^1 \log g_{t-1} + \rho_g^2 \log g_{t-2} - (1 - \rho_g^1 - \rho_g^2) \gamma_{gb} (by_{t-1} - by) + \varepsilon_{g,t}$$
(46)

$$\tau_{c,t} = (1 - \rho_c^1 - \rho_c^2)\tau_c + \rho_c^1\tau_{c,t-1} + \rho_c^2\tau_{c,t-2} + (1 - \rho_c^1 - \rho_c^2)\gamma_{cb}(by_{t-1} - by) + \varepsilon_{c,t}$$
(47)
$$\tau_{c,t} = (1 - \rho_c^1 - \rho_c^2)\tau_c + \rho_c^1\tau_{c,t-1} + \rho_c^2\tau_{c,t-2} + (1 - \rho_c^1 - \rho_c^2)\gamma_{cb}(by_{t-1} - by) + \varepsilon_{c,t}$$
(47)

$$\tau_{w,t} = (1 - \rho_w^1 - \rho_w^2)\tau_w + \rho_w^1\tau_{w,t-1} + \rho_w^2\tau_{w,t-2} + (1 - \rho_w^1 - \rho_w^2)\gamma_{wb}(by_{t-1} - by) + \varepsilon_{w,t}$$
(48)

$$\tau_{K,t} = (1 - \rho_K^2 - \rho_K)\tau_K + \rho_K^2\tau_{K,t-1} + \rho_K^2\tau_{K,t-2} + (1 - \rho_K^2 - \rho_K)\tau_K b (by_{t-1} - by) + \varepsilon_{K,t}$$
(43)
$$\tau_t = (1 - \rho_\tau^1 - \rho_\tau^2)\tau + \rho_\tau^1\tau_{t-1} + +\rho_\tau^2\tau_{t-2} + (1 - \rho_\tau^1 - \rho_\tau^2)\gamma_{\tau b} (by_{t-1} - by) + \varepsilon_{\tau,t}$$
(50)

$$\log \zeta_t = \rho_\zeta \log \zeta_{t-1} + \varepsilon_{\zeta,t} \tag{51}$$

$$\log \zeta_t^L = \rho_L \log \zeta_{t-1}^L + \varepsilon_{L,t} \tag{52}$$

$$\log \zeta_t^I = \rho_I \log \zeta_{t-1}^I + \varepsilon_{I,t}$$
(53)
$$\zeta_t^b = (1 - \varepsilon_t) \zeta_t^b + \varepsilon_t \zeta_t^b + \varepsilon_t \zeta_t^b$$
(54)

$$\zeta_t^b = (1 - \rho_b)\zeta^b + \rho_b\zeta_{t-1}^b + \varepsilon_{b,t} \tag{54}$$

A.3 Steady State

$$\frac{z - h\beta}{c(z - h)} = \lambda(1 + \tau_c) \tag{55}$$

$$r^F = \frac{z}{\beta} \tag{56}$$

$$r = \frac{z}{\beta} \tag{57}$$

$$p^B = \frac{(1 + \kappa^B p^B)}{r} \tag{58}$$

$$(1 - \tau_K)r^K = \frac{z}{\beta} + \delta - 1 \tag{59}$$

$$\lambda = \phi \tag{60}$$

$$l^{\nu} = \frac{\lambda(1 - \tau_w)w}{\gamma} \tag{61}$$

$$i = k - (1 - \delta) \frac{k}{z} \tag{62}$$

$$y = k^{\alpha} \left(l\right)^{1-\alpha} z^{-\alpha} \tag{63}$$

$$r^{K} = \alpha \frac{gz}{k} \tag{64}$$

$$w = \frac{(1-\alpha)y}{l} \tag{65}$$

$$nx = y - c - i - g \tag{66}$$

$$ca = nx + (r^F - 1)\frac{b^T}{z}$$
(67)

$$b^F = \frac{r^F}{z}b^F + nx \tag{68}$$

$$r^F = r^* \exp(\zeta^b) \tag{69}$$

$$g + \left(\frac{(1+\kappa^B p^B)}{z} - p_B\right)b = \tau_c c + \tau_w w l + \frac{\tau_K r^K k}{z} + \tau$$
(70)

A.4 Log-Linear Equations and Observation Equations

Tax rates, debt, foreign debt, the current account and trade balance are defined in terms of deviations from their steady state values. For these variables we use the notation, $\tilde{x}_t = x_t - x$. Remaining variables are expressed in log-deviations, that is $\hat{x}_t = \log x_t - \log x$.

A.4.1 Structural Equations

$$0 = (z - h\beta)(z - h)\left(\hat{\lambda}_t + \frac{1}{1 + \tau_c}\tilde{\tau}_{c,t}\right) + (z^2 + h^2\beta)\hat{c}_t - h\beta z\hat{c}_{t+1} - hz\hat{c}_{t-1} + hz\hat{z}_t - h\beta z\hat{z}_{t+1} - (z - h)(z\hat{\zeta}_t - h\beta\hat{\zeta}_{t+1})$$
(71)

$$0 = -\hat{\lambda}_t + \hat{\lambda}_{t+1} + \hat{r}_t - \hat{z}_{t+1}$$
(72)

$$0 = -\hat{\lambda}_t + \hat{\lambda}_{t+1} + \hat{r}_t^F - \hat{z}_{t+1}$$
(73)

$$0 = \hat{p}_t^B - \frac{\kappa^B}{(1 + \kappa^B p^B)} \hat{p}_{t+1}^B + \hat{r}_t$$
(74)

$$0 = -\hat{\phi}_t + \frac{\beta(1-\tau_K)r^K}{z} \left(\hat{\lambda}_{t+1} - \hat{z}_{t+1} + \hat{r}_{t+1}^K\right) - \frac{\beta r^K}{z} \tilde{\tau}_{K,t+1} + \frac{\beta(1-\delta)}{z} \left(\hat{\phi}_{t+1} - \hat{z}_{t+1}\right)$$
(75)

$$0 = -\hat{\lambda}_t + \hat{\phi}_t + \hat{\zeta}_t^I - z^2 \Upsilon'' \left((1+\beta)\hat{i}_t - \hat{i}_{t-1} - \beta\hat{i}_{t+1} + \hat{z}_t - \beta\hat{z}_{t+1} \right)$$
(76)

$$0 = \hat{\lambda}_t + \hat{w}_t - \frac{1}{1 - \tau^w} \tilde{\tau}_t^w - \nu \hat{l}_t - \hat{\zeta}_t^L - \hat{\zeta}_t$$
(77)

$$0 = \hat{k}_t - \frac{1 - \delta}{z} (\hat{k}_{t-1} - \hat{z}_t) - \frac{z - 1 + \delta}{z} (\hat{i}_t + \hat{\zeta}_t^I)$$
(78)

$$0 = \hat{y}_t - \alpha \hat{k}_{t-1} - (1 - \alpha)\hat{l}_t + \alpha \hat{z}_t$$
(79)

$$0 = \hat{y}_t + \hat{z}_t - \hat{k}_{t-1} - \hat{r}_t^K \tag{80}$$

$$0 = \hat{y}_t - \hat{l}_t - \hat{w}_t \tag{81}$$

$$0 = -\tilde{n}x_t + y\hat{y}_t - c\hat{c}_t - i\hat{i}_t - g\hat{g}_t$$
(82)

$$0 = -\tilde{c}a_t + \tilde{n}x_t + \frac{r^F b^F}{z} \hat{r}_{t-1}^F + \left(\frac{r^F - 1}{z}\right) \tilde{b}_{t-1}^F - \left(\frac{r^F - 1}{z}\right) b^F \hat{z}_t$$
(83)

$$0 = -\tilde{b}_{t}^{F} + \tilde{n}x_{t} + \frac{r^{F}}{z}\tilde{b}_{t-1}^{F} + \frac{r^{F}b^{F}}{z}\hat{r}_{t-1}^{F} - \frac{r^{F}b^{F}}{z}\hat{z}_{t}$$
(84)

$$0 = -\hat{r}_{t}^{F} + \hat{r}_{t}^{*} - \frac{\psi_{b}}{y}\tilde{b}_{t}^{F} + \frac{\psi_{b}b^{F}}{y}\hat{y}_{t} + \zeta_{t}^{b}$$
(85)

$$0 = g\hat{g}_{t} + \frac{\kappa^{B}p^{B}b}{z}\hat{p}_{t}^{B} + \frac{(1+\kappa^{B}p^{B})}{z}\tilde{b}_{t-1} - \frac{(1+\kappa^{B}p^{B})b}{z}\hat{z}_{t} - p^{B}\tilde{b}_{t} - p^{B}b\hat{p}_{t}^{B} - c\tilde{\tau}_{c,t} - \tau_{c}c\hat{c}_{t} - wl\tilde{\tau}_{w,t} - wl\tau_{w}(\hat{w}_{t} + \hat{l}_{t}) - \frac{r^{K}k}{z}\tilde{\tau}_{K,t} - \frac{\tau_{K}r^{K}k}{z}(\hat{r}_{t}^{K} + \hat{k}_{t-1} - \hat{z}_{t}) - \tilde{\tau}_{t}$$
(86)

$$0 = -\hat{g}_t + \rho_g^1 \hat{g}_{t-1} + \rho_g^2 \hat{g}_{t-2} - (1 - \rho_g^1 - \rho_g^2) \gamma_{gb} \left(\frac{p^B}{y} \tilde{b}_{t-1} + \frac{p^B b}{y} \hat{p}_{t-1}^B - \frac{p^B b}{y} \hat{y}_{t-1}\right) + \varepsilon_{g,t}$$
(87)

$$0 = -\tilde{\tau}_{c,t} + \rho_c^1 \tilde{\tau}_{c,t-1} + \rho_c^2 \tilde{\tau}_{c,t-2} + (1 - \rho_c^1 - \rho_c^2) \gamma_{cb} \left(\frac{p^B}{y} \tilde{b}_{t-1} + \frac{p^B b}{y} \hat{p}_{t-1}^B - \frac{p^B b}{y} \hat{y}_{t-1}\right) + \varepsilon_{c,t}$$
(88)

$$0 = -\tilde{\tau}_{w,t} + \rho_w^1 \tilde{\tau}_{w,t-1} + \rho_w^2 \tilde{\tau}_{w,t-2} + (1 - \rho_w^1 - \rho_w^2) \gamma_{wb} \left(\frac{p^B}{y} \tilde{b}_{t-1} + \frac{p^B b}{y} \hat{p}_{t-1}^B - \frac{p^B b}{y} \hat{y}_{t-1}\right) + \varepsilon_{w,t}$$
(89)

$$0 = -\tilde{\tau}_{K,t} + \rho_K^1 \tilde{\tau}_{K,t-1} + \rho_K^2 \tilde{\tau}_{K,t-2} + (1 - \rho_K^1 - \rho_K^2) \gamma_{Kb} \left(\frac{p^B}{y} \tilde{b}_{t-1} + \frac{p^B b}{y} \hat{p}_{t-1}^B - \frac{p^B b}{y} \hat{y}_{t-1}\right) + \varepsilon_{K,t} \quad (90)$$

$$0 = -\tilde{\tau}_t + \rho_\tau^1 \tilde{\tau}_{t-1} + \rho_\tau^2 \tilde{\tau}_{t-2} + (1 - \rho_\tau^1 - \rho_\tau^2) \gamma_{\tau b} \left(\frac{p^B}{y} \tilde{b}_{t-1} + \frac{p^B b}{y} \hat{p}_{t-1}^B - \frac{p^B b}{y} \hat{y}_{t-1}\right) + \varepsilon_{\tau,t}$$
(91)

$$0 = \hat{z}_t - \rho_z \hat{z}_{t-1} - \varepsilon_{z,t} \tag{92}$$

$$0 = \hat{r}_t^* - \rho_{r^*} \hat{r}_{t-1}^* - \varepsilon_{r^*,t}$$
(93)

$$0 = \hat{\zeta}_t - \rho_\zeta \hat{\zeta}_{t-1} - \varepsilon_{\zeta,t} \tag{94}$$

$$0 = \hat{\zeta}_t^L - \rho_L \hat{\zeta}_{t-1}^L - \varepsilon_{L,t} \tag{95}$$

$$0 = \hat{\zeta}_t^I - \rho_I \hat{\zeta}_{t-1}^I - \varepsilon_{I,t} \tag{96}$$

$$0 = \zeta_t^b - (1 - \rho_b)\zeta^b - \rho_b \zeta_{t-1}^b - \varepsilon_{b,t}$$

$$\tag{97}$$

A.4.2 Observation Equations

$$\Delta \hat{y}_t^{obs} = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{98}$$

$$\Delta \hat{c}_t^{obs} = \hat{c}_t - \hat{c}_{t-1} + \hat{z}_t \tag{99}$$

$$\frac{\hat{g}^{obs}}{y_t} = \hat{g}_t - \hat{y}_t \tag{100}$$

$$\tilde{nx}_t^{obs} = \frac{1}{y}\tilde{nx}_t - \frac{nx}{y}\hat{y}$$
(101)

$$\Delta \hat{w}_t^{obs} = \hat{w}_t - \hat{w}_{t-1} + \hat{z}_t \tag{102}$$

$$\hat{r}_t^{obs} = \hat{r}_t \tag{103}$$

$$\hat{r}^{*obs}_{t} = \hat{r^{*}}_{t} \tag{104}$$

$$\left(\frac{p^{\tilde{B}}b}{y}\right)_{t}^{obs} = \frac{p^{B}}{y}\tilde{b}_{t} + \frac{p^{B}b}{y}\hat{p}_{t}^{B} - \frac{p^{B}b}{y}\hat{y}_{t}$$
(105)

$$\left(\frac{\hat{\tau_c c}}{y}\right)_t^{oos} = \frac{c}{y}\tilde{\tau}_{c,t} + \frac{\tau_c c}{y}\hat{c}_t - \frac{\tau_c c}{y}\hat{y}_t$$
(106)

$$\left(\frac{\tau_w wl}{y}\right)_t^{oos} = \frac{wl}{y}\tilde{\tau}_{w,t} + \frac{\tau_w wl}{y}\hat{w}_t + \frac{\tau_w wl}{y}\hat{l}_t - \frac{\tau_w wl}{y}\hat{y}_t$$
(107)

$$\left(\frac{\tau_K \hat{r}^K k}{y}\right)_t^{obs} = \frac{r^K k}{y} \tilde{\tau}_{K,t} + \frac{\tau_K r^K k}{y} \hat{r}_t^K + \frac{\tau_K r^K k}{y} \hat{k}_{t-1} - \frac{\tau_K r^K k}{y} \hat{y}_t$$
(108)

.

A.5 Derivation of Life-time Utility Function

The representative household maximises expected lifetime utility given by:

$$\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \zeta_{t} \left(\log \left(C_{t} - h C_{t-1} \right) - \zeta_{t}^{L} \frac{L_{t}^{1+\nu}}{1+\nu} \right)$$
(109)

Using the normalised consumption variable $c_t = \frac{C_t}{Z_t}$ and the fact that $Z_t = z_t Z_{t-1}$, the utility function can be written as:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left(\log(Z_t) + \log\left(c_t - h\frac{c_{t-1}}{z_t}\right) - \zeta_t^L \frac{L_t^{1+\nu}}{1+\nu} \right)$$
(110)

Along the balanced growth path, and using $\ln(Z_t) = t \ln(z)$, we have:

$$U = \sum_{t=0}^{\infty} \beta^{t} \left(\log(c) - \frac{L^{1+\nu}}{1+\nu} + \log\left(1 - \frac{h}{z}\right) + t\log(z) \right)$$

= $\frac{1}{1-\beta} \left[\log(c) - \frac{L^{1+\nu}}{1+\nu} + \log\left(1 - \frac{h}{z}\right) + \frac{\beta\log(z)}{1-\beta} \right]$ (111)

B Data Sources

This section describes the data used to estimate the model.

- **Population:** Quarterly gross domestic product in chain volume measure (ABS Catalogue 5206.001) divided by quarterly gross domestic product per capita also in chain volume measure (ABS Catalogue 5206.001).
- Real GDP per capita: Quarterly gross domestic product per capita in chain volume measure (ABS Catalogue 5206.001). This series enters in first difference in the estimation.
- Consumption per capita: Quarterly private consumption in chain volume measure (ABS Catalogue 5206.002) divided by population. This series enters in first difference in the estimation with its sample mean adjusted to match the sample mean of real output growth.
- Government spending-to-GDP ratio: Quarterly government consumption and pubic gross fixed capital formation in current prices (ABS Catalogue 5206.003) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003). This series enters in log form in the estimation.
- Net exports-to-GDP ratio: Net exports-to-GDP is computed as exports-to-GDP less imports-to-GDP. Exports-to-GDP is quarterly exports in current price measure divided by quarterly gross domestic product in current prices. Imports to-GDP is quarterly imports in current prices divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003). The sample mean of this series is removed prior to the estimation.
- Hourly wage: Compensation of employees (ABS Cat 5206.044) divided by the hours worked index (ABS Cat 5206.001). The series is deflated by the consumption deflator (ABS Cat 5206.005). This series enters in first difference with its sample mean adjusted to equal the mean of output growth.
- Domestic Real interest rate: 90-day bank bill rate (RBA Bulletin Table F1). This nominal interest rate is converted to a real rate using the trimmed mean inflation series (RBA Bulletin Table G1). The monthly series is converted into quarterly frequency by arithmetic averaging.
- Foreign Real interest rate: 3-months U.S. Treasury bill rate (FRED Database). This nominal interest rate is converted to a real rate using the U.S. core PCE inflation series (FRED Database). The monthly series is converted into quarterly frequency by arithmetic averaging.

- Government debt-to-GDP-ratio: Commonwealth government securities on issue (Australian Office of Financial Management and RBA Bulletin Table E3) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003).
- Consumption tax revenues-to-GDP-ratio: The sum of sales tax revenues and goods and services tax revenues in current prices (ABS Cat 5206.022) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003). The mean of the series is adjusted for the subsample 1983-2000 to adjust for the break resulting from the introduction of the goods and services tax in the year 2000.
- Labour income tax revenues-to-GDP ratio: Individual income tax revenues in current prices (ABS Cat 5206.022) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003).
- Capital income tax revenues-to-GDP ratio: The sum of resident corporations' income tax revenues and non-residents' income tax revenues in current prices (ABS Cat 5206.022) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003)

C Solution and Estimation Procedure

The model is estimated and solved using the technique developed in Kulish and Pagan (2017) for models with structural changes. We allow for the structural breaks in steady-state trend growth, z, steady-state labour income tax rate, τ_w , steady-state capital income tax rate τ_K and in the variance of shocks to happen at possibly different dates in the sample, T_z and T_σ . Hence, for the data sample $t = 1, 2, \dots, T$, and assuming that $T_z < T_\sigma$, three different regimes occur:

1. First regime: For $t = 1, 2, \dots, T_z - 1$, steady-state labour-augmenting technology growth takes an initial value, z. In the initial regime, the first-order approximation to the equilibrium conditions around the steady state is a linear rational expectations system of equations that is given by:

$$A_0 y_t = C_0 + A_1 y_{t-1} + \mathbb{E}_t B_0 y_{t+1} + D_0 \varepsilon_t$$
(112)

where the structural matrices A_0 , C_0 , A_1 , B_0 and D_0 correspond to the initial steady state, y_t is vector of state and jump variables and ε_t is a vector of exogenous *iid* shocks. The solution, if it exists and is unique, will be a Vector Autoregression (VAR) that takes the form:

$$y_t = C + Qy_{t-1} + G\varepsilon_t \tag{113}$$

2. Second regime: For $t = T_z, \dots, T_{\sigma} - 1$, steady-state labour-augmenting technology growth takes a different value, say z'. The structural form of the model then becomes:

$$A_0^* y_t = C_0^* + A_1^* y_{t-1} + \mathbb{E}_t B_0^* y_{t+1} + D_0 \varepsilon_t$$
(114)

where the superscript * is associated with the matrices that correspond to the new steadystate commodity price level. Note that the matrix D_0 is unchanged as the break in the variances of shocks hasn't occurred yet. The solution, if it exists and is unique, will be a VAR that takes the form:

$$y_t = C^* + Q^* y_{t-1} + G^* \varepsilon_t$$
 (115)

3. Third regime: For $t = T_{\sigma}, \dots, T$, the variances of shocks change. The structural form of the model then becomes:

$$A_0^* y_t = C_0^* + A_1^* y_{t-1} + \mathbb{E}_t B_0^* y_{t+1} + D_0^{**} \varepsilon_t$$
(116)

where the matrix D_0^{**} denotes the matrix corresponding to the new variances of shocks while other structural matrices are maintained as in the second regime. The solution, if it exists and is unique, will be a VAR that takes the form:

$$y_t = C^* + Q^* y_{t-1} + G^{**} \varepsilon_t \tag{117}$$

Based on the three regimes, the time-varying reduced form is given by:

$$y_t = C_t + Q_t y_{t-1} + G_t \varepsilon_t \tag{118}$$

Given a data sample, one can form an observable variables vector, y_t^{obs} , that relates to the variables in the model by:

$$y_t^{obs} = Hy_t + v_t \tag{119}$$

where v_t is a vector of *iid* measurement errors with zero mean and covariance matrix V. Together, the state equation, Equation (118), and the observation equation, Equation (119), form a state-space model. Hence, the data sample's likelihood function can be constructed by using the Kalman filter as outlined in Kulish and Pagan (2017).

To estimate the model, the standard practice in the DSGE literature is to use Bayesian techniques that place informative priors on the estimated parameters. Bayesian methods are adopted to estimate non-calibrated parameters (ϑ) and the dates of structural changes (**T**). In this framework, a prior distribution on non-calibrated parameters and dates of structural changes, $p(\vartheta, \mathbf{T})$, is updated by sample information contained in the likelihood function $\mathcal{L}(Y|\vartheta, \mathbf{T})$ to form a posterior distribution

$$p(\vartheta, \mathbf{T}|Y) = \mathcal{L}(Y|\vartheta, \mathbf{T})p(\vartheta, \mathbf{T})$$
(120)

Since the mapping from the DSGE model to its likelihood function $\mathcal{L}(Y|\vartheta, \mathbf{T})$ is nonlinear in the parameters, the construction of the posterior distribution is too complicated to evaluate analytically. Therefore, the Metropolis Hastings algorithm is used to simulate from the joint posterior distribution of the structural parameters and the date breaks.

C.1 Computation of the non-stochastic transition paths

The initial regime has a steady state given by $y = (I - Q)^{-1}C$. When there is a break in trend growth or in other parameters, the steady state would shift to $y^* = (I - Q^*)^{-1}C^*$. We compute the non-stochastic transition path from one steady state to another, as the path the economy takes in the absence of business cycle shocks, that is $\varepsilon_t = 0$ for all t, but in the presence of the regime changes. This path is given by the time-varying VAR with $\varepsilon_t = 0$, that is the path given by $y_t = C_t + Q_t y_{t-1}$.

D Growth Accounting Calculations

To perform the growth accounting exercise, we assume Australia's output per capita can be modelled as a Cobb-Douglas aggregate of available technology and capital per capita:

$$y_t = A_t k_t^{\alpha} \tag{121}$$

where y_t is output per capita, A_t is total factor productivity, and k_t is capital per capita. Hence, output per capita growth, g_y , is given as:

$$g_y = g_a + \alpha g_k \tag{122}$$

where g_a is the contribution of total factor productivity to output per capita growth and αg_k is the contribution of capital per capita of output growth. The results of the growth accounting calculations for Australia are given in Table D.1.

Period	Average GDP per capita growth %	Contribution of capital per capita $\%$	Contribution of total factor productivity %
1990-2000	2.02	0.61	1.41
1990-2017	1.65	0.70	0.95
2000-2017	1.36	0.77	0.59
2010-2017	1.10	0.70	0.40

Table D.1: Growth Accounting Calculations for Australia

Below is a description of the data used in the growth accounting calculation:

- **Population:** Annual gross domestic product in chain volume measure (ABS Catalogue 5204.0) divided by annual gross domestic product per capita also in chain volume measure (ABS Catalogue 5204.0).
- **Real GDP per capita:** Gross domestic product using the production based approach in chain volume measure (ABS Catalogue 5204.0) divided by population.
- Capital per capita: End-year net capital stock in chain volume measure (ABS catalogue 5204.0) divided by population.
- Capital share in production function: The ratio of gross operating surplus in all sectors to income. Income is computed as the sum of compensation of employees (ABS Catalogue 5204.0) and gross operating surplus in all sectors (ABS Catalogue 5204.0).

E Unobserved Components Estimates

We set up linear and non-linear unobserved components trend-cycle decomposition models for the quarterly level of GDP and allow for a break in output trend to happen at any date as well as a break in the variance of the shock to the trend and variance of the shock to the cycle to occur on the same date.

E.1 Linear Unobserved Components Model

The linear unobserved components trend-cycle decomposition model is given by:

$$y_t = \tau_t + c_t \tag{123}$$

$$\tau_t = z\mathbf{1}(t < T_z) + (z + \Delta z)\mathbf{1}(t \ge T_z) + \tau_{t-1} + \epsilon_t^{\tau}$$
(124)

$$c_t = \rho_1 c_{t-1} + \rho_2 c_{t-2} + \epsilon_t^c \tag{125}$$

where y_t is the logarithm of Australia's real GDP per capita which is decomposed into a trend component τ_t and a cyclical component c_t . The trend component τ_t is specified as a random walk with a drift and we allow for a break in the drift to happen at the date T_z . $\mathbf{1}(A)$ is an indicator function that takes the value 1 if the condition A is true and a value of 0 otherwise. As such, the mean growth rate of the trend equals z before the break date T_z , and $z' = z + \Delta z$ on and after the break date. The cyclical component c_t is modelled as a zero-mean stationary AR(2) process. We assume that the innovations ϵ_t^{τ} and ϵ_t^{c} are independently normal:

$$\begin{pmatrix} \epsilon_t^{\tau} \\ \epsilon_t^{c} \end{pmatrix} = \mathcal{N} \left(\mathbf{0}, \begin{bmatrix} \mu \sigma_{\tau}^2 \mathbf{1}(t < T_{\sigma}) + \sigma_{\tau}^2 \mathbf{1}(t \ge T_{\sigma}) & 0 \\ 0 & \mu \sigma_c^2 \mathbf{1}(t < T_{\sigma}) + \sigma_c^2 \mathbf{1}(t \ge T_{\sigma}) \end{bmatrix} \right)$$

We allow for a break in the variances of the innovations ϵ_t^{τ} and ϵ_t^c to occur at the same date T_{σ} . As such, the variances of the shocks to the trend and the cycle are respectively $\mu \sigma_{\tau}^2$ and $\mu \sigma_c^2$ before the break date T_{σ} , and σ_{τ}^2 and σ_c^2 on and after the break date.

The linear unobserved components trend-cycle decomposition model can be written is state space form:

$$y_t = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x_t \tag{126}$$

$$x_{t} = \begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} \mathbf{1}(t < T_{z}) + \begin{bmatrix} z' \\ 0 \\ 0 \end{bmatrix} \mathbf{1}(t \ge T_{z}) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_{1} & \rho_{2} \\ 0 & 1 & 0 \end{bmatrix} x_{t-1} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{t}^{\mathsf{T}} \\ \epsilon_{t}^{\mathsf{C}} \end{bmatrix}$$
(127)

where $x_t = \begin{bmatrix} \tau_t & c_t & c_{t-1} \end{bmatrix}'$.

To estimate the model, we calibrate the growth rate in the initial regime at 0.0055 as in the small open economy model and use a Bayesian estimation technique to estimate the remaining parameters (ϑ) and the break dates (**T**). We set the priors to either be in consistence with

the literature or to be uninformative. Uniform prior with support -0.0045 to 0.015 is set for the mean growth of the trend parameter z'. Normal distribution with mean 0.9 and standard deviation 1 is imposed on the autoregressive parameter ρ_1 . For the autoregressive parameter ρ_2 , we impose a normal prior with mean 0 and standard deviation 1. The priors on the standard deviations of shocks, σ_{τ} and σ_c are set as uniform priors with support 0 and 0.2. Further, a uniform prior [0, 3] is imposed on the variance scale parameter μ . Finally, flat priors are imposed for the break date T_z and T_{σ} and the initial regime is restricted to be at least 60 quarters long. The prior and posterior distributions of the parameters from estimating the model at level and at first-difference are listed in Tables E.1 and E.2, respectively.

	Prie	or distribu	ution		Posterior d	listribution	l
Parameter	Dist.	Mean	S.d.	Mean	Mode	5%	95%
Parameter	rs						
z'	Uniform	[-0.00]	45, 0.015]	0.0025	0.0029	0.0014	0.0036
$ ho_1$	Normal	0.9	1	0.8344	0.8907	0.1365	1.4520
ρ_2	Normal	0	1	-0.2197	0.0017	-0.7982	0.4072
σ_{τ}	Uniform	[0]	[0.2]	0.0074	0.0079	0.0031	0.0094
σ_c	Uniform	[0]	,0.2	0.0026	0.0004	0.0002	0.0076
μ	Uniform	[(0,3]	1.9941	1.8537	1.5998	2.4486
T_z	Flat	L	(, 2015:Q2]	2006:Q3	2008:Q1	2002:Q2	2008:Q4
$\tilde{T_{\sigma}}$	Flat		, 2015:Q2]	2002:Q1	2004:Q2	1998:Q2	2005:Q2

 Table E.1: Prior and Posterior Distribution of the Parameters and Break Dates from Level

 Estimation

Table E.2: Prior and Posterior Distribution of the Parameters and Break Dates from First-Difference Estimation

	Pri	or distribu	tion	Posterior distribution							
Parameter	Dist.	Mean	S.d.	Mean	Mode	5%	95%				
Parameter	rs										
z'	Uniform	[-0.004]	45, 0.015]	0.0025	0.0030	0.0014	0.0036				
$ ho_1$	Normal	0.9	1	0.8237	0.9620	0.1286	1.4446				
ρ_2	Normal	0	1	-0.2154	0.0000	-0.8041	0.4214				
$\sigma_{ au}$	Uniform	[0,	0.2]	0.0074	0.0001	0.0029	0.0096				
σ_c	Uniform	[0,	0.2]	0.0026	0.0078	0.0002	0.0076				
μ	Uniform	[0	[,3]	2.0051	1.8420	1.6041	2.4713				
T_z	Flat	[1997:Q4	, 2015:Q2]	2006:Q3	2008:Q1	2002:Q2	2008:Q4				
T_{σ}	Flat	[1997:Q4	, 2015:Q2]	2002:Q1	2004:Q2	1998:Q2	2005:Q2				

E.2 Non-linear Unobserved Components Model

The non-linear unobserved components model is set up as a Friedman's Plucking model as in Kim and Nelson (1999). Here, the trend-cycle decomposition is given by:

$$y_t = \tau_t + c_t \tag{128}$$

$$\tau_t = z \mathbf{1} (t < T_z) + (z + \Delta z) \mathbf{1} (t \ge T_z) + \tau_{t-1} + \epsilon_t^{\tau}$$
(129)

$$c_t = \rho_1 c_{t-1} + \rho_2 c_{t-2} + \pi_{S_t} + \epsilon_t^c \tag{130}$$

$$\pi_{S_t} = \pi S_t, \qquad \pi \neq 0 \tag{131}$$

where π_{S_t} is an asymmetric, discrete, shock which is dependent upon an unobserved variable S_t . We assume that S_t evolves according to a first-order Markov-switching process as in Hamilton (1989):

$$Pr[S_t = 1|S_{t-1} = 1] = p \tag{132}$$

$$Pr[S_t = 0|S_{t-1} = 0] = q \tag{133}$$

As in the linear model, the trend component τ_t is specified as a random walk with a drift and we allow for a break in the drift to happen at the date T_z . The non-linear unobserved components trend-cycle decomposition model can be written is state space form:

$$y_{t} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x_{t}$$

$$x_{t} = \begin{bmatrix} z \\ \pi_{S_{t}} \\ 0 \end{bmatrix} \mathbf{1}(t < T_{z}) + \begin{bmatrix} z' \\ \pi_{S_{t}} \\ 0 \end{bmatrix} \mathbf{1}(t \ge T_{z}) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_{1} & \rho_{2} \\ 0 & 1 & 0 \end{bmatrix} x_{t-1} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{t}^{\tau} \\ \epsilon_{t}^{c} \end{bmatrix}$$
(135)

where $x_t = \begin{bmatrix} \tau_t & c_t & c_{t-1} \end{bmatrix}'$.

To estimate the non-linear model, we calibrate the growth rate in the initial regime at 0.0055 as in the small open economy model and use a Bayesian estimation technique to estimate the remaining parameters (ϑ) and the break dates (**T**). In estimation, the Kim (1994) filter is used which combines the Kalman filter with Hamilton (1989) filter for Markov-switching models. The prior and posterior distributions of the parameters from estimating the model at level are listed in Table E.3.

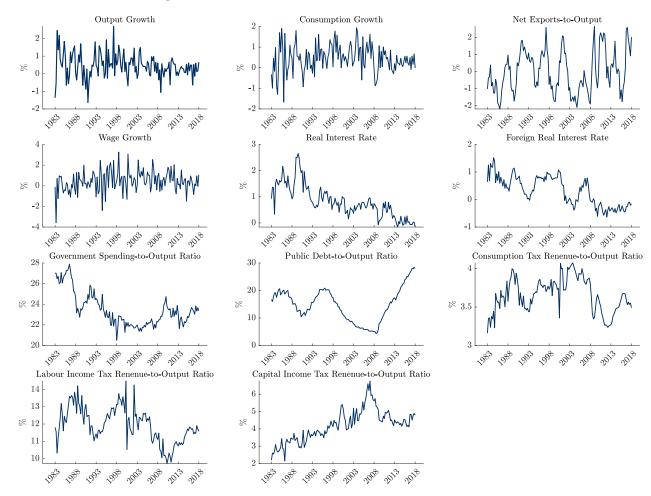
Table E.3: Prior and Posterior Distribution of the Parameters and Break Dates from Level Estimation

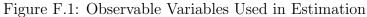
	Prie	or distrib	ution		Posterior d	listribution	1
Parameter	Dist.	Mean	S.d.	Mean	Mode	5%	95%
Parameter	rs						
z'	Uniform	[-0.00]	45, 0.015]	0.0025	0.0029	0.0015	0.0035
$ ho_1$	Normal	0.9	1	1.0420	0.5306	0.2979	1.5729
$ ho_2$	Normal	0	1	-0.3338	0.0004	-0.7660	0.1852
$\sigma_{ au}$	Uniform	[0	, 0.2]	0.0072	0.0079	0.0057	0.0092
σ_c	Uniform	[0	, 0.2]	0.0004	0.0028	0.0003	0.0061
μ	Uniform	[[[0, 3]	2.0993	1.9003	1.6438	2.6442
π	Uniform	[-0.0]	[05, 0.05]	0.0015	-0.0015	-0.0041	0.0063
p	Beta	0.05	0.15	0.0504	0.0000	0.0026	0.1446
q	Beta	0.25	0.1	0.2605	0.2241	0.1196	0.4275
T_z	Flat	$[1997:Q_{2}]$	4, 2015:Q2]	2006:Q2	2007:Q2	2001:Q3	2008:Q3
T_{σ}	Flat	$[1997:Q_{2}]$	4, 2015:Q2]	2002:Q1	2004:Q1	1998:Q1	2008:Q1

F Additional Estimation Results

F.1 Observables

Figure F.1 plots the observables used in the model estimation: the growth rate of GDP per capita in chain volume terms, the growth rate of private consumption per capita in chain volume terms, government spending as a share of GDP, net-exports as a share of GDP, the growth rate of the real hourly wage, the domestic real interest rate, the foreign real interest rate, public debt as a share of GDP, as well as consumption tax revenues, labour income tax revenues, and capital income tax revenues as shares of GDP.





Sources: ABS; AOFM; Authors' calculations; FRED; RBA

F.2 Posterior Distribution of Trend Growth and Date Breaks

Figure F.2 shows the posterior distribution of $z' = z + \Delta z$ together with the mean of growth for the period 1983-2008 which is our calibrated value for trend growth in the initial regime. After the break, trend growth in GDP per capita in annual terms is estimated to be around 0.93% at the mode of the posterior. And while there is some uncertainty around this estimate, there is little mass close to the trend growth rate of the initial regime.

Figure F.2: Posterior Distribution of Trend Growth

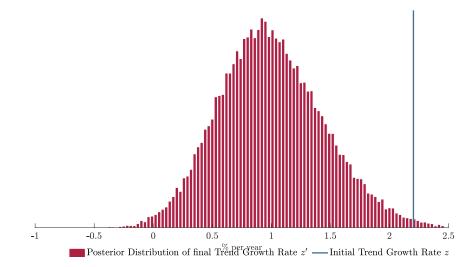
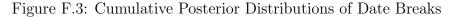
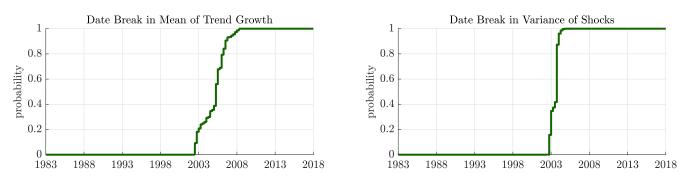


Figure F.3 shows the estimated cumulative distribution functions for the date breaks in trend growth and the variance of shocks. The mean for the break in trend growth is estimated to be the fourth quarter of 2004 while the mode is the first quarter of 2005. There is about 60% probability that the break in trend growth occurred between 2003 and 2005; the remaining 40% probability is spread between 2005 and 2008.





F.3 Prior and Posterior Distributions of Parameters

Figure F.4 plots the probability density functions under the prior and the posterior. All posterior densities are estimated using the Epanechnikov kernel function. Even with diffuse priors applied to the majority of parameters, the data proves to be highly informative, steering the posteriors toward more concentrated regions within the parameter space.

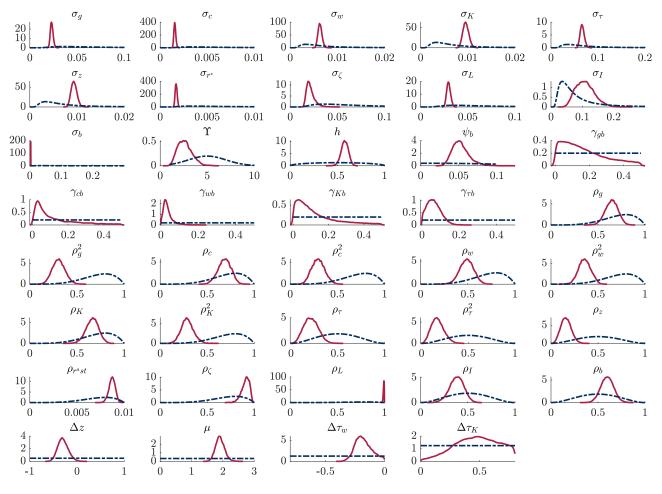


Figure F.4: Prior and Posterior Distributions

---- Prior Distribution —— Posterior Distribution

F.4 Variance Decompositions

In our sample, the economy can be in one of four possible regimes. The estimated cumulative distribution functions, however, suggest that the most prevalent are the high trend growth high variance and the low trend growth low variance regimes. Table F.1 computes variance decompositions of the two regimes for the observable series used in estimation.

In spite of the estimated regime changes, the contributions of shocks to the variance of the observables is broadly stable across regimes. Productivity and labour supply shocks account for over 80 per cent of the variance of output growth. Fiscal policy shocks, shocks to government spending and tax revenues, however, do not account for the bulk of the fluctuations in output growth, consumption growth, net exports, wage growth and the domestic real interest rates which suggests that fiscal policy is not a significant source of macroeconomic volatility.

					~						
					S	hock					
Variable	c	6 D.	6.	G.r.	6.	61	c	c	c	6	c
Initial Regime	ε_z	ε_{R^*}	ε_{ζ}	ε_L	ε_I	ε_b	ε_g	ε_c	ε_w	ε_K	$\varepsilon_{ au}$
Output growth	39.6	2.4	7.0	36.4	4.7	3.4	1.5	0.0	4.9	0.1	0.0
Consumption growth	17.5	3.4	30.9	27.6	11.7	2.8	4.8	$0.0 \\ 0.1$	0.4	0.1	0.0
Net exports/GDP	2.7	13.2	11.0	6.2	56.4	5.8	3.5	0.0	1.0	$0.0 \\ 0.2$	0.0
Wage growth	77.5	0.7	3.1	11.9	3.7	1.0	0.0	0.0	1.6	0.2 0.1	0.0
Domestic real interest rate	1.6	19.3	7.9	2.0	24.3	43.4	0.9	0.0	0.2	$0.1 \\ 0.4$	0.0
Foreign real interest rate	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0
Government spending/GDP	0.8	0.0	1.4	60.3	1.1	0.0	36.1	0.0	0.0	0.0	0.0
Government debt/GDP	0.0	0.0	0.7	44.8	0.5	0.0	31.5	6.3	2.5	5.2	8.0
Consumption tax revenues/GDP	0.5	0.1	1.7	15.1	0.7	0.0	4.5	75.6	0.3	0.9	0.6
Labour income tax revenues/GDP	0.2	0.0	0.4	27.1	0.3	0.0	17.9	3.9	43.5	2.9	3.7
Capital income tax revenues/GDP	0.1	0.0	0.2	14.6	0.1	0.0	8.0	2.1	0.5	73.1	1.3
Final Regime	20.7	9.4	7.0	971	4 1	3.4	1.3	0.0	4 7	0.1	0.0
Output growth	39.7	2.4	7.0	37.1	4.1				4.7	0.1	0.0
Consumption growth	18.0	3.5	31.5	27.2	11.5	2.8	4.4	0.1	0.4	0.5	0.0
Net exports/GDP	2.9	14.2	12.9	6.9	51.9	6.2	3.8	0.0	1.0	0.2	0.0
Wage growth	78.0	0.7	3.1	12.1	3.0	1.0	0.4	0.0	1.5	0.1	0.0
Domestic real interest rate	1.5	19.9	8.4	2.1	22.1	44.6	0.9	0.0	0.2	0.4	0.0
Foreign real interest rate	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Government spending/GDP	0.9	0.0	1.4	60.6	1.0	0.0	35.8	0.0	0.1	0.2	0.0
Government debt/GDP	0.5	0.0	0.8	45.8	0.5	0.0	30.3	6.9	2.5	5.3	7.6
Consumption tax revenues/GDP	0.5	0.1	1.8	16.1	0.7	0.0	4.3	74.7	0.3	0.9	0.6
Labour income tax revenues/GDP	0.3	0.0	0.5	28.0	0.3	0.0	17.4	4.2	42.9	3.0	3.6
Capital income tax revenues/GDP	0.1	0.0	0.2	15.2	0.1	0.0	7.8	2.3	0.5	72.5	1.3

Table F.1: Variance Decompositions

Note: The variance shares are reported in per cent.

G Additional Model Validation

G.1 Prior Predictive Analysis

We evaluate the performance of the estimated model by comparing the model's predictions under the prior and posterior densities with the actual data. We take 100 draws from the prior and posterior densities and at each draw compute the non-stochastic transition path. Figure G.1 compares the paths under both the prior and posterior densities of the observables. The predictions of the observable variables from the prior density exhibit significant deviations from the empirical paths. Meanwhile, the posterior density shrinks that uncertainty and matches the actual data better. This suggests that the model's parameter estimates obtained through estimation improve the model's non-stochastic transitional dynamics.

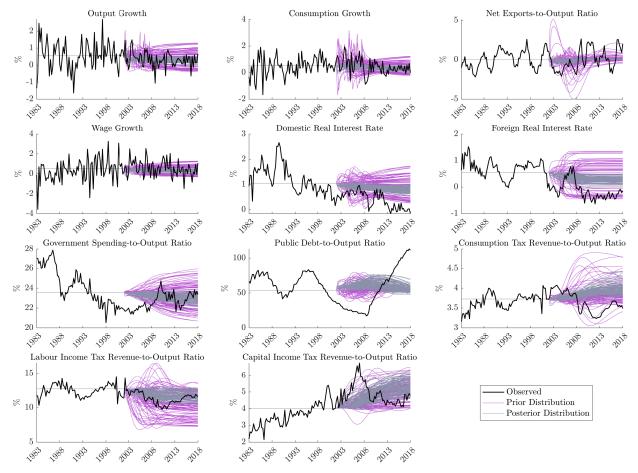


Figure G.1: Transitional Dynamics from Prior and Posterior Densities

Sources: ABS; Authors' calculations; FRED; RBA

We also assess the variability of the model variables. Table G.1 compares the theoretical standard deviations of the observables used in estimation computed at the posterior mode and the prior mode values of the parameters with their empirical counterparts. Table G.1 also

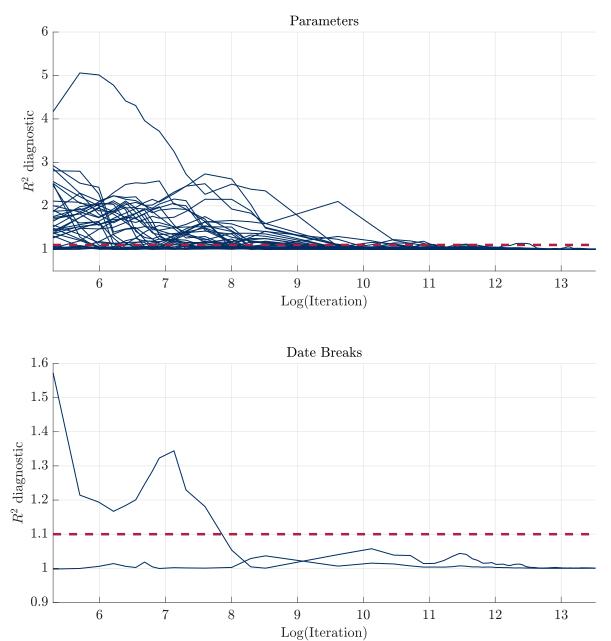
reports the 90% probability interval implied by the model's posterior and prior densities. It is evident that for most variables, the mode of the posterior standard deviations is notably closer to the standard deviation of the data than the mode of the prior standard deviations. This is particularly true for wage growth, the domestic real interest rate, the foreign real interest rate, government spending-to-output ratio and capital income tax revenue-to-output ratio where the posterior modes are closer to their empirical counterpart. The 90% confidence intervals for the posterior standard deviations are also noticeably narrower, indicating a more precise estimate. This shows that the model estimation leads to improvement in the model's fit.

	Data	Poster	rior distri	bution	Prio	r distribu	ition
	[1983 - 2018]	Mode	5%	95%	Mode	5%	95%
Output growth	0.0069	0.0113	0.0113	0.0130	0.0568	0.0253	0.1740
Consumption growth	0.0066	0.0103	0.0097	0.0116	0.1273	0.0097	0.2914
Net exports/GDP	0.0123	0.0210	0.0201	0.0281	0.1154	0.0401	0.3690
Wage growth	0.0101	0.0096	0.0094	0.0111	0.0238	0.0123	0.0747
Domestic real interest rate	0.0061	0.0064	0.0062	0.0077	0.1266	0.0313	0.2802
Foreign real interest rate	0.0053	0.0054	0.0049	0.0071	0.0041	0.0044	0.0619
Government spending/GDP	0.0678	0.0798	0.0608	0.1337	0.1696	0.1097	0.8845
Public debt/GDP	0.2419	0.6123	0.3392	0.8516	0.1392	0.1295	1.2614
Consumption tax revenues/GDP	0.0022	0.0081	0.0041	0.0137	0.0128	0.0063	0.0715
Labour income tax revenues/GDP	0.0103	0.0119	0.0091	0.0215	0.0202	0.0127	0.1302
Capital income tax revenues/GDP	0.0090	0.0103	0.0078	0.0258	0.0042	0.0045	0.0509

Table G.1: Standard Deviations from Prior and Posterior Densities

G.2 MCMC Chain Convergence

In Figure G.2, we present the convergence analysis of the two chains executed during the estimation process, employing the Gelman et al. (2019) R^2 diagnostic. The R^2 diagnostic lies below the threshold value of 1.1 for all parameters and date breaks by the end of the chain. This indicated convergence across chains in the posterior distributions, validating the reliability of the estimation results.





G.3 Investment Shocks and Credit Spreads

Figure G.3 illustrates the correlation between the filtered innovation of the marginal efficiency of investment and the Australian non-financial corporate A-rated bonds' spread to swap across various tenors. The figure reveals a negative correlation between the filtered investment efficiency innovation and credit spreads at different leads and lags. This is consistent with economic intuition. Improved investment efficiency tends to coincide with lower credit spreads, reflecting positive economic conditions and reduced credit risk.

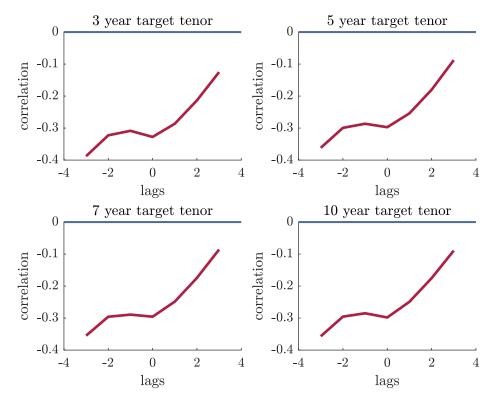
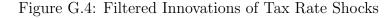


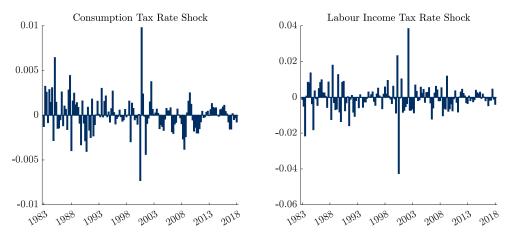
Figure G.3: Correlation (credit spread(t), marginal efficiency of investment(t - j))

Sources: Authors' calculations; RBA

G.4 Tax Shocks and the GST reform of 2000

Figure G.4 illustrates the filtered innovations in the consumption tax rate shock and the labour income tax rate shock evaluated at the mode of the posterior distribution. The abrupt surge in the shock to the tax rate on consumption aligns with the implementation of the Goods and Services Tax (GST) in Australia in mid-2000. This tax reform also entailed a decrease in personal income tax, which is also captured by the downward spike in the innovation to the labor income tax rate. Consequently, the estimated tax shocks are consistent in tax policy changes in Australia.





G.5 Testing for Structural Breaks

In the model estimation, we allow for a single break in trend growth. To explore the potential for multiple structural breaks in the time series, we conduct the Bai and Perron (1998) test for multiple structural changes in linear models on quarterly Australian real GDP per capita growth spanning the period 1983:Q1-2018:Q1. The results are presented in Table G.2. The findings reveal that the hypothesis of zero breaks is rejected in favour of at least one break. However, when comparing one break against two or more breaks, the test fails to reject the null hypothesis. Consequently, we conclude that there is one break in the series of real GDP per capita growth, aligning with our assumption in the estimation. Furthermore, the estimated break date is identified as 2007:Q3, consistent with the outcomes obtained from the unobserved components model. Thus, the results of the Bai and Perron (1998) test lend support to the assumption of a single break in the trend growth in the structural model.

Null	Alternative	Test Statistic	5% Critical Value
H0: no breaks	H1: 1 break	6.97	5.74
H0: 1 break	H1: 2 breaks	2.03	6.47
H0: 2 breaks	H1: 3 breaks	2.55	7.01
H0: 3 breaks	H1: 4 breaks	7.69	7.42
H0: 4 breaks	H1: 5 breaks	2.41	7.64

Table G.2: Bai-Perron Test Results

G.6 Results from Alternative Specifications

		Base	eline		Ν	o Change i	n Tax Rat	es
Parameter	Mean	Mode	5%	95%	Mean	Mode	5%	95%
Structural	Paramet	ers						
h	0.58	0.57	0.51	0.64	0.58	0.59	0.52	0.65
$\Upsilon^{''}$	2.62	2.45	1.51	3.99	2.67	2.41	1.56	3.98
ψ_b	0.53	0.50	0.37	0.74	0.52	0.50	0.37	0.72
Δz	-0.0030	-0.0032	-0.0047	-0.0010	-0.0026	-0.0026	-0.0042	-0.0009
$\Delta \tau_w$	-0.0179	-0.0201	-0.0284	-0.0047				
$\Delta \tau_K$	0.0474	0.0485	0.0158	0.0759				
μ	1.91	1.89	1.68	2.16	1.91	1.93	1.68	2.17
Fiscal Rul								
γ_{gb}	0.166	0.026	0.013	0.418	0.166	0.036	0.012	0.421
γ_{cb}	0.120	0.029	0.013	0.380	0.103	0.026	0.011	0.348
γ_{wb}	0.037	0.021	0.007	0.091	0.113	0.057	0.016	0.342
γ_{Kb}	0.124	0.032	0.009	0.374	0.142	0.052	0.013	0.392
$\gamma_{\tau b}$	0.064	0.053	0.010	0.131	0.066	0.050	0.013	0.131
	0.64	0.65	0.53	0.75	0.65	0.65	0.53	0.76
ρ_c^1	0.68	0.70	0.56	0.79	0.69	0.69	0.57	0.80
ρ_w^1	0.49	0.49	0.37	0.61	0.55	0.55	0.44	0.66
ρ_K^1	0.66	0.67	0.55	0.77	0.67	0.67	0.55	0.77
ρ_{τ}^{1}	0.21	0.19	0.09	0.34	0.21	0.18	0.09	0.33
ρ_a^2	0.31	0.31	0.20	0.42	0.30	0.30	0.19	0.42
ρ_c^2	0.30	0.29	0.19	0.41	0.29	0.27	0.18	0.40
ρ_w^2	0.36	0.36	0.25	0.47	0.40	0.39	0.29	0.51
ρ_K^2	0.29	0.28	0.18	0.40	0.29	0.29	0.19	0.40
$\rho_{q}^{1} \\ \rho_{c}^{1} \\ \rho_{w}^{1} \\ \rho_{K}^{1} \\ \rho_{K}^{1} \\ \rho_{q}^{2} \\ \rho_{c}^{2} \\ \rho_{w}^{2} \\ \rho_{K}^{2} \\ \rho_{\tau}^{2} \\ \rho_{\tau$	0.17	0.16	0.07	0.29	0.17	0.15	0.07	0.28
Other AR	Coefficie							
$ ho_z$	0.16	0.15	0.07	0.25	0.16	0.16	0.07	0.25
$ ho_{R^*}$	0.87	0.87	0.81	0.92	0.88	0.88	0.83	0.93
$ ho_{\zeta}$	0.91	0.92	0.83	0.96	0.92	0.94	0.85	0.98
$ ho_L$	0.99	0.99	0.98	1.00	0.99	1.00	0.98	1.00
$ ho_I$	0.38	0.40	0.25	0.50	0.38	0.39	0.26	0.50
$ ho_b$	0.60	0.60	0.48	0.71	0.60	0.61	0.48	0.71
Standard 2	Deviation	s						
σ_z	0.009	0.009	0.008	0.010	0.009	0.009	0.008	0.010
σ_{R^*}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
σ_{ζ}	0.022	0.019	0.016	0.030	0.025	0.020	0.017	0.039
σ_L	0.030	0.029	0.026	0.033	0.030	0.030	0.027	0.034
σ_I	0.109	0.108	0.065	0.163	0.108	0.095	0.066	0.156
σ_b	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
σ_g	0.023	0.023	0.021	0.026	0.023	0.023	0.021	0.026
σ_c	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.002
σ_w	0.006	0.006	0.006	0.007	0.006	0.006	0.006	0.007
σ_K	0.010	0.010	0.009	0.011	0.010	0.010	0.009	0.011
$\sigma_{ au}$	0.067	0.066	0.059	0.074	0.066	0.066	0.059	0.074
Date Brea	\mathbf{ks}							
T_z	2004:Q4	2005:Q1	2002:Q2	2007:Q2	2004:Q4	2005:Q1	2002:Q2	2007:Q2
T_{σ}	2003:Q3	2003:Q3	2002:Q3	2003:Q4	2003:Q3	2003:Q3	2002:Q3	2003:Q4
Log margi	nal likelik	nood	-5651.5			-564	45.0	-

Table G.3: Results from Specification with No Changes in Tax Rates

		Bas	eline		Trai	nsitory Tec	hnology Sh	ocks
Parameter	Mean	Mode	5%	95%	Mean	Mode	5%	95%
Structura	al Paramet	ers						
h	0.58	0.57	0.51	0.64	0.57	0.58	0.51	0.64
$\Upsilon^{''}$	2.62	2.45	1.51	3.99	3.11	2.66	1.88	4.64
ψ_b	0.53	0.50	0.37	0.74	0.52	0.48	0.36	0.73
Δz	-0.0030	-0.0032	-0.0047	-0.0010	-0.0025	-0.0023	-0.0045	-0.0004
$\Delta \tau_w$	-0.0179	-0.0201	-0.0284	-0.0047	-0.0176	-0.0204	-0.0277	-0.0039
$\Delta \tau_K$	0.0474	0.0485	0.0158	0.0759	0.0498	0.0556	0.0175	0.0767
μ	1.91	1.89	1.68	2.16	1.92	1.91	1.69	2.16
Fiscal Ru	ıles Param	eters						
γ_{gb}	0.166	0.026	0.013	0.418	0.134	0.014	0.008	0.405
γ_{cb}	0.120	0.029	0.013	0.380	0.117	0.026	0.012	0.426
γ_{wb}	0.037	0.021	0.007	0.091	0.042	0.027	0.007	0.104
γ_{Kb}	0.124	0.032	0.009	0.374	0.121	0.016	0.009	0.364
$\gamma_{\tau b}$	0.064	0.053	0.010	0.131	0.063	0.050	0.009	0.133
ρ_q^1	0.64	0.65	0.53	0.75	0.64	0.63	0.51	0.76
$ ho_c^1$	0.68	0.70	0.56	0.79	0.69	0.69	0.58	0.80
$ ho_w^1$	0.49	0.49	0.37	0.61	0.48	0.47	0.36	0.60
$ ho_K^1$	0.66	0.67	0.55	0.77	0.66	0.65	0.54	0.77
$ ho_{ au}^1$	0.21	0.19	0.09	0.34	0.21	0.21	0.09	0.34
$ ho_g^2$	0.31	0.31	0.20	0.42	0.29	0.29	0.17	0.40
$ ho_c^2$	0.30	0.29	0.19	0.41	0.29	0.29	0.18	0.40
$ \begin{array}{c} \rho_{1}^{1} \\ \rho_{c}^{1} \\ \rho_{w}^{1} \\ \rho_{K}^{1} \\ \rho_{\tau}^{1} \\ \rho_{c}^{2} \\ \rho_{c}^{2} \\ \rho_{w}^{2} \\ \rho_{K}^{2} \\ \rho_{\tau}^{2} \end{array} $	0.36	0.36	0.25	0.47	0.36	0.35	0.25	0.48
$ ho_K^2$	0.29	0.28	0.18	0.40	0.29	0.28	0.18	0.40
$ ho_{ au}^2$	0.17	0.16	0.07	0.29	0.17	0.16	0.07	0.29
Other AI	R Coefficie							
$ ho_z$	0.16	0.15	0.07	0.25	0.56	0.61	0.27	0.81
$ ho_a$					0.94	0.97	0.86	0.98
$ ho_{R^*}$	0.87	0.87	0.81	0.92	0.88	0.88	0.82	0.93
$ ho_{\zeta}$	0.91	0.92	0.83	0.96	0.91	0.94	0.82	0.97
$ ho_L$	0.99	0.99	0.98	1.00	0.99	0.99	0.98	1.00
$ ho_I$	0.38	0.40	0.25	0.50	0.32	0.34	0.19	0.45
$ ho_b$	0.60	0.60	0.48	0.71	0.59	0.57	0.47	0.71
	Deviation							
σ_z	0.009	0.009	0.008	0.010	0.003	0.003	0.002	0.005
σ_a	0.000	0.000	0.000	0.000	0.006	0.006	0.005	0.007
σ_{R^*}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
σ_{ζ}	0.022	0.019	0.016	0.030	0.023	0.020	0.016	0.035
σ_L	0.030	0.029	0.026	0.033	0.030	0.029	0.026	0.033
σ_I	0.109	0.108	0.065	0.163	0.131	0.117	0.082	0.191
σ_b	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
σ_g	0.023	0.023	0.021	0.026	0.020	0.020	0.018	0.023
σ_c	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.002
σ_w	0.006	0.006	0.006	0.007	0.006	0.006	0.006	0.007
σ_K	0.010	0.010	0.009	0.011	0.010	0.010	0.009	0.011
σ_{τ}	0.067	0.066	0.059	0.074	0.066	0.066	0.059	0.074
Date Bre		2005-01	2002-02	2007-02	2004.04	2005-01	2002-02	9007.01
T_z	2004:Q4	2005:Q1	2002:Q2	2007:Q2	2004:Q4	2005:Q1	2002:Q2	2007:Q1
T_{σ}	2003:Q3	2003:Q3	2002:Q3	2003:Q4	2003:Q1	2003:Q3	2002:Q3	2003:Q4

Table G.4: Results from Specification with Transitory Technology Shock

H Additional Figures

H.1 Data for Small Open Economies

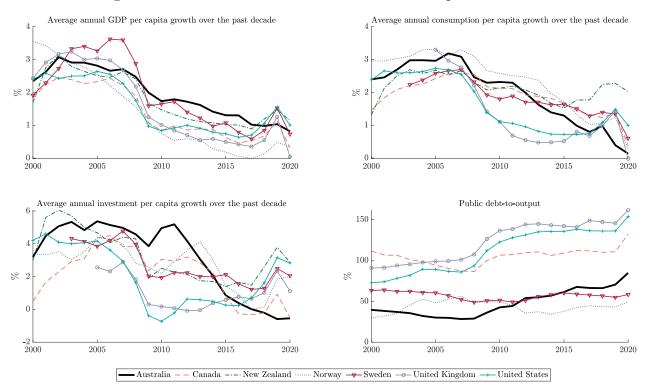
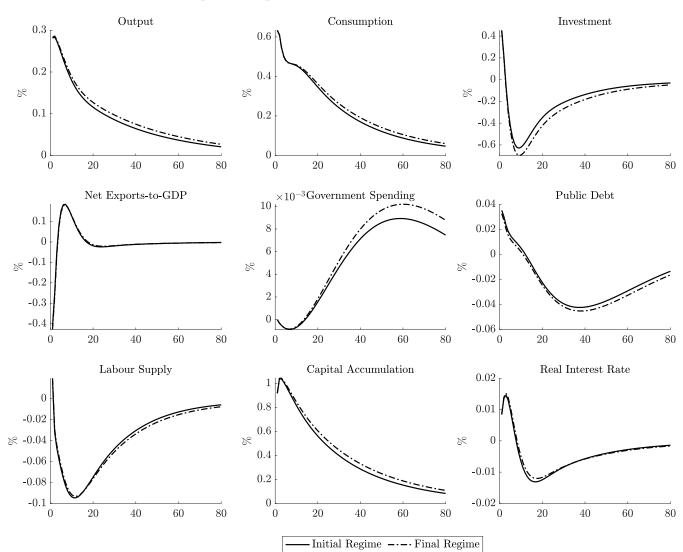


Figure H.1: Macroeconomic Outcomes for Small Open Economies

Sources: ABS; Authors' calculations; FRED; RBA



H.2 Impulse Responses to Trend Growth Shock

Impulse Responses to Trend Growth Shock

H.3 Alternative Fiscal Responses to Change in Trend Growth

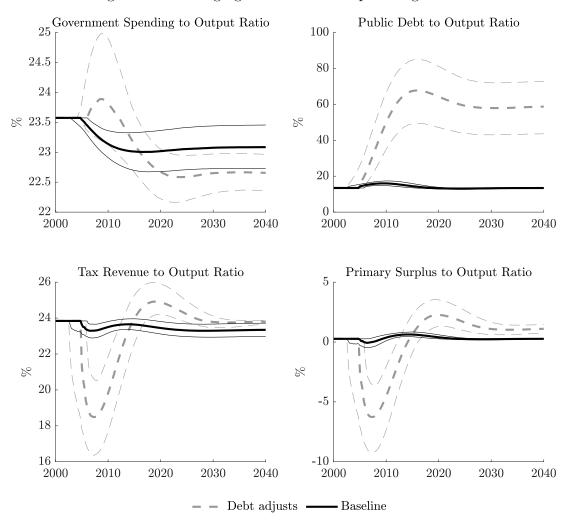


Figure H.2: Changing the Debt to Output Target Ratio

Note: The solid-dark lines show the median transition path and its 80% confidence band at the estimated posterior distribution. The dashed-grey lines show the counterfactual transition path and its 80% confidence band when only the debt-to-output ratio changes to satisfy the government's budget constraint.

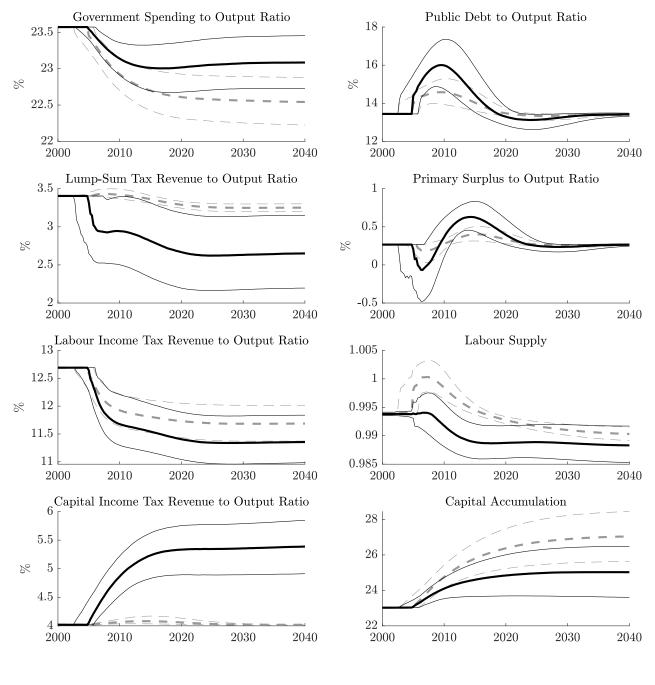
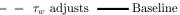


Figure H.3: Labour Income Tax Rate Adjusts



Note: The solid-dark lines show the median transition path and its 80% confidence band at the estimated posterior distribution. The dashed-grey lines show the counterfactual transition path and its 80% confidence band when only the labour income tax rate changes to satisfy the government's budget constraint.

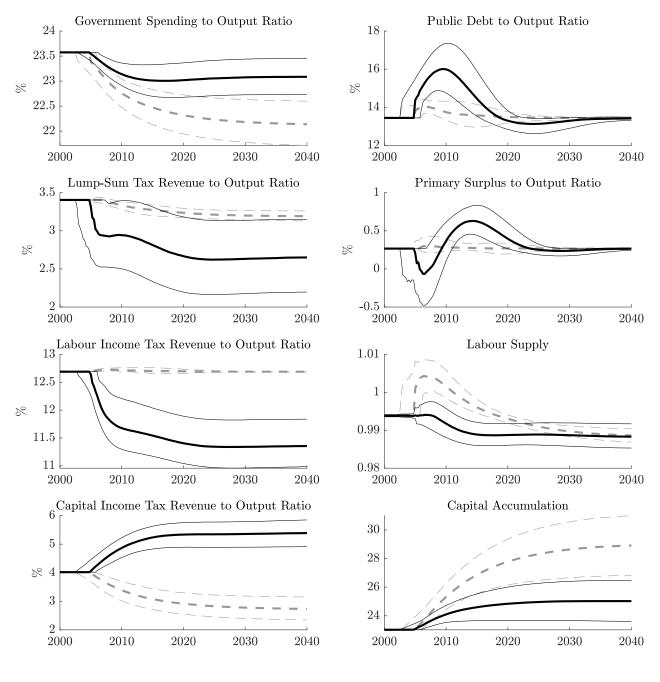


Figure H.4: Capital Income Tax Rate Adjusts

 $- - \tau_K$ adjusts — Baseline

Note: The solid-dark lines show the median transition path and its 80% confidence band at the estimated posterior distribution. The dashed-grey lines show the counterfactual transition path and its 80% confidence band when only the capital income tax rate changes to satisfy the government's budget constraint.

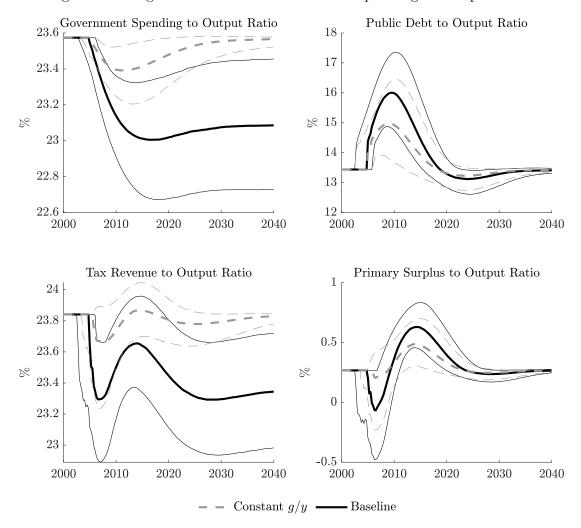


Figure H.5: Figure 9: Constant Government Spending to Output Ratio

Note: The solid-dark lines show the median transition path and its 80% confidence band at the estimated posterior distribution. The dashed-grey lines show the counterfactual transition path and its 80% confidence band when the government maintains the g/y ratio across balanced growth paths.

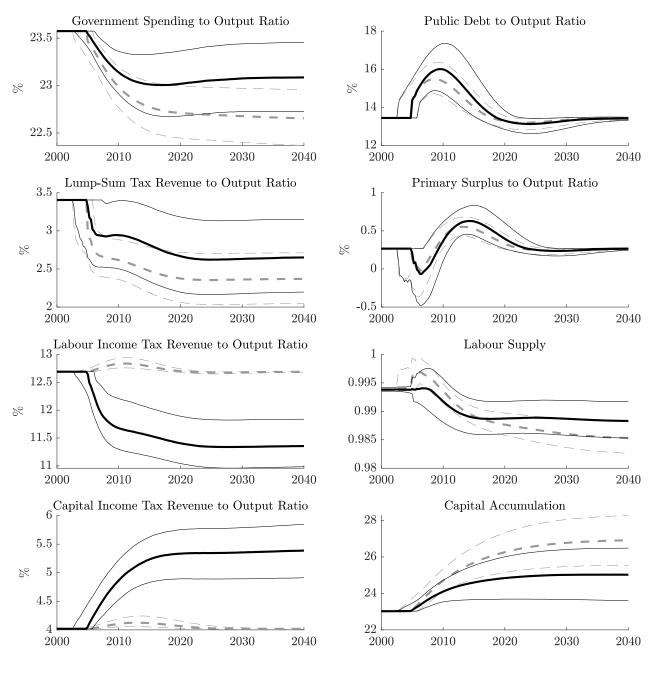


Figure H.6: Lump-Sum Taxes Adjust

– – Lump-sum taxes adjust – Baseline

Note: The solid-dark lines show the median transition path and its 80% confidence band at the estimated posterior distribution. The dashed-grey lines show the counterfactual transition path and its 80% confidence band when only lump-sum taxes change to satisfy the government's budget constraint.

H.4 Transitional Dynamics from Alternative Model Specifications

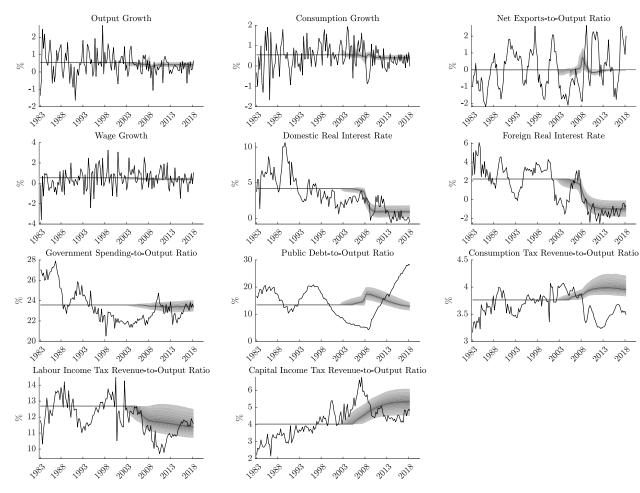


Figure H.7: Estimated Transitional Dynamics from Model with Financial Frictions

Net Exports-to-Output Ratio

Foreign Real Interest Rate

198⁹

MM

Sources: ABS; Authors' calculations; FRED; RBA

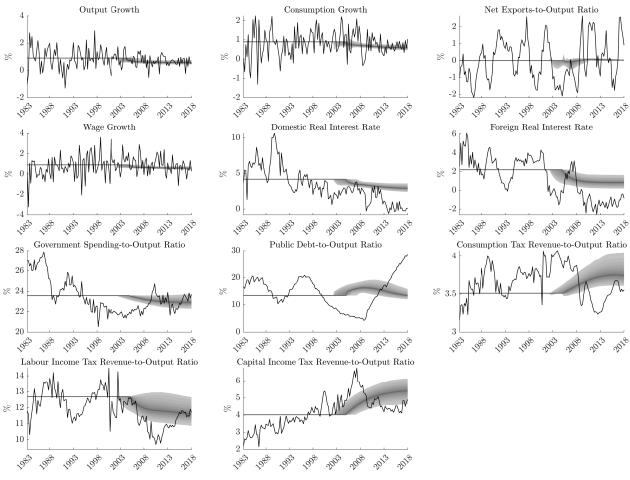


Figure H.8: Estimated Transitional Dynamics from Model with Population Growth

Sources: ABS; Authors' calculations; FRED; RBA

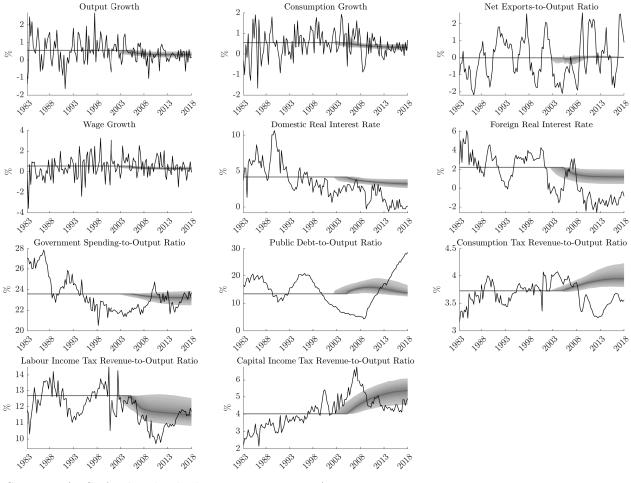


Figure H.9: Estimated Transitional Dynamics from Model with Transitory Productivity Shock

Sources: ABS; Authors' calculations; FRED; RBA

I Model with Financial Frictions and Population Growth

I.1 Households and Firms

The representative households of size N_t maximises expected lifetime utility given by:

$$\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} N_{t}^{1-\theta} \left(\log \left(\tilde{c}_{t} - h \tilde{c}_{t-1} \right) - \zeta_{t}^{L} \frac{\tilde{l}_{t}^{1+\nu}}{1+\nu} + \chi_{b} Z_{t}^{-1} \left(\tilde{b}_{t} + \tilde{b}_{t}^{F} \right) \right)$$
(136)

where $\tilde{c}_t = C_t/N_t$ is consumption per person and $\tilde{l}_t = L_t/Nt$ are hours worked per person, $\tilde{b}_t = B_t/N_t$ are government bonds per person and $\tilde{b}_t^F = B_t^F/N_t$ are foreign bonds per person. The parameter $h \in [0, 1]$ governs the degree of external habit formation, $1/\nu$ is the Frisch elasticity and χ_b determines the convenience yield. ζ_t is an intertemporal preference shock that follows:

$$\log \zeta_t = \rho_\zeta \log \zeta_{t-1} + \varepsilon_{\zeta,t} \tag{137}$$

and ζ_t^L is a labour supply shock that follows:

$$\log \zeta_t^L = \rho_L \log \zeta_{t-1}^L + \varepsilon_{L,t} \tag{138}$$

The growth rate of population $n_t = N_t/N_{t-1}$ is subject to stochastic shocks $\varepsilon_{n,t}$ and evolves as follows:

$$\log n_t = (1 - \rho_n) \log n + \rho_n \log n_{t-1} + \varepsilon_{n,t}$$
(139)

Following Becker and Barro (1988), the parameter θ represents the weighting factor with respect to the household size N_t . With $\theta = 0$, the per-capita utility of each generation is weighted by its size (Benthamite preferences). With $\theta = 1$ the per-capita utility of each generation is weighted equally, regardless of its size (Millian preferences).

The flow budget constraint of the household is

$$(1+\tau_{c,t})C_t + I_t + P_t^B B_t + B_t^F \le (1+\kappa^B P_t^B)B_{t-1} + R_{t-1}^F B_{t-1}^F + (1-\tau_{w,t})W_t L_t + (1-\tau_{K,t})r_t^K K_{t-1} + TR_t + C_{t-1}^F R_t^F + C_{t-1}^F R$$

Here, C_t is consumption, $\tau_{c,t}$ is the tax rate on consumption, I_t is investment, B_t stands for government bonds and P_t^B is the price of government bonds, B_t^F stands for foreign bonds and R_t^F for its gross rate of return, L_t are hours worked, W_t is the real wage per hour worked and $\tau_{w,t}$ is the tax rate on labour income. The capital stock available for production at time t is K_{t-1} and r_t^K is its rental rate, while $\tau_{K,t}$ is the tax rate on capital income. TR_t stands for lump sum taxes or transfers.

The budget constraint expressed in per-capita terms is:

$$(1+\tau_{c,t})\tilde{c}_{t}+\tilde{i}_{t}+P_{t}^{B}\tilde{b}_{t}+\tilde{b}_{t}^{F} \leq (1+\kappa^{B}P_{t}^{B})\frac{N_{t-1}}{N_{t}}\tilde{b}_{t-1}+R_{t-1}^{F}\frac{N_{t-1}}{N_{t}}\tilde{b}_{t-1}^{F} + (1-\tau_{w,t})W_{t}\tilde{l}_{t}+(1-\tau_{K,t})r_{t}^{K}\frac{N_{t-1}}{N_{t}}\tilde{k}_{t-1}+\tilde{t}r_{t}$$

$$(140)$$

where lower case letters with a \sim denote per-capita quantities.

The capital stock per person evolves according to

$$\tilde{k}_t = (1-\delta) \frac{N_{t-1}}{N_t} \tilde{k}_{t-1} + \zeta_t^I \left[1 - \Upsilon \left(\frac{\tilde{i}_t}{\tilde{i}_{t-1}} \right) \right] \tilde{i}_t$$
(141)

The function that governs the investment adjustment cost satisfies, $\Upsilon(z) = \Upsilon'(z) = 0$ and $\Upsilon'' > 0$. ζ_t^I is a shock to the marginal efficiency of investment which is assumed to follow:

$$\log \zeta_t^I = \rho_I \log \zeta_{t-1}^I + \varepsilon_{I,t} \tag{142}$$

Output is produced with a Cobb-Douglas production function by competitive firms hiring capital and labour:

$$Y_{t} = K_{t-1}^{\alpha} \left(Z_{t} L_{t} \right)^{1-\alpha}$$
(143)

where Z_t is labour-augmenting technology whose growth rate, $z_t = Z_t/Z_{t-1}$, follows:

$$\log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + \varepsilon_{z,t}$$
(144)

and so z governs the growth rate of labour-augmenting TFP along the balanced growth path.

I.2 Trade Balance and Net Foreign Assets

The interest rate that the household receives on foreign bonds depends on the economy's net foreign asset position according to the debt-elastic interest rule:

$$R_t^F = R_t^* \exp\left[-\psi_b \left(\frac{b_t^F}{y_t} - \frac{b^F}{y}\right) + \zeta_t^b\right]$$
(145)

where $\frac{b^F}{y}$ is the steady-state ratio of net foreign assets to output, and ζ_t^b is the country risk premium shock which follows the process below:

$$\zeta_t^b = (1 - \rho_b)\zeta^b + \rho_b \zeta_{t-1}^b + \varepsilon_{b,t}$$
(146)

and R_t^* is the foreign real interest rate which follows the exogenous process below:

$$\log R_t^* = (1 - \rho_{R^*}) \log R^* + \rho_{R^*} \log R_{t-1}^* + \varepsilon_{R^*,t}$$
(147)

The trade balance is output less domestic absorption, that is,

$$NX_{t} = Y_{t} - C_{t} - I_{t} - G_{t} \tag{148}$$

and the current account is therefore given by:

$$CA_t = NX_t + (R_{t-1}^F - 1)B_{t-1}^F$$
(149)

In equilibrium, net foreign assets evolve according to:

$$B_t^F = R_{t-1}^F B_{t-1}^F + NX_t (150)$$

Per capita variables, except for hours worked and interest rates, trend at the rate of z. When normalised by Z_t , however, the variables $b_t = \tilde{b}_t/Z_t = B_t/(N_tZ_t)$, $c_t = \tilde{c}_t/Z_t = C_t/(N_tZ_t)$, $y_t = \tilde{y}_t/Z_t = Y_t/(N_tZ_t)$, and so on, converge in the absence of shocks to their steady state values which we denote by b, c, y and so on.

I.3 The Government

The government receives tax payments on consumption, labour and capital income as well as lump-sum taxes and borrows domestically to finance government spending. Thus, the government budget constraint is:

$$P_t^B B_t + \tau_{c,t} C_t + \tau_{w,t} W_t L_t + \tau_{K,t} r_t^K K_{t-1} + T R_t = (1 + \kappa^B P_t^B) B_{t-1} + G_t$$
(151)

We assume the government sets government spending and taxes rates following fiscal rules which include a response to deviations of the government debt to output ratio from its steady state. In particular, we assume rules of the form:

$$\log g_t = (1 - \rho_g^1 - \rho_g^2) \log g + \rho_g^1 \log g_{t-1} + \rho_g^2 \log g_{t-2} - (1 - \rho_g^1 - \rho_g^2) \gamma_{gb} (by_{t-1} - by) + \varepsilon_{g,t}$$
(152)

$$\tau_{c,t} = (1 - \rho_c^1 - \rho_c^2)\tau_c + \rho_c^1\tau_{c,t-1} + \rho_c^2\tau_{c,t-2} + (1 - \rho_c^1 - \rho_c^2)\gamma_{cb}(by_{t-1} - by) + \varepsilon_{c,t}$$
(153)

$$\tau_{w,t} = (1 - \rho_w^1 - \rho_w^2)\tau_w + \rho_w^1 \tau_{w,t-1} + \rho_w^2 \tau_{w,t-2} + (1 - \rho_w^1 - \rho_w^2)\gamma_{wb} \left(by_{t-1} - by\right) + \varepsilon_{w,t}$$
(154)

$$\tau_{K,t} = (1 - \rho_K^1 - \rho_K^2)\tau_K + \rho_K^1 \tau_{K,t-1} + \rho_K^2 \tau_{K,t-2} + (1 - \rho_K^1 - \rho_K^2)\gamma_{Kb} (by_{t-1} - by) + \varepsilon_{K,t}$$
(155)

$$\tau_t = (1 - \rho_\tau^1 - \rho_\tau^2)\tau + \rho_\tau^1 \tau_{t-1} + \rho_\tau^2 \tau_{t-2} + (1 - \rho_\tau^1 - \rho_\tau^2)\gamma_{\tau b} (by_{t-1} - by) + \varepsilon_{\tau,t}$$
(156)

where the normalised variables $\tau_t = \frac{TR_t}{N_t Z_t}$, $y_t = \frac{Y_t}{N_t Z_t}$, $g_t = \frac{G_t}{N_t Z_t}$, $b_t = \frac{B_t}{N_t Z_t}$, have steady states τ, y, g and b respectively. Throughout, τ is set so that given all other fiscal policy rule parameters the government budget constraint, equation (151), holds in steady state.

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