Securitization of Subprime Credit and the Propagation of Housing Shocks

Online Appendix

May, 2023

1 Background on the U.S. Subprime Mortgage Market

Subprime mortgages are residential loans granted to individuals who do not qualify for prime mortgages since they do not conform to the lenders' underwriting standards. Subprime mortgage loans developed as a new class of loans in the mid 1990's with the intention of allowing riskier borrowers to access mortgage finance in order to facilitate the expansion of home ownership in the U.S. economy (Gorton, 2008). In fact, the U.S. housing policy has been focused for decades on expanding home ownership opportunities for low-income households, and subprime mortgages were the financial innovation aimed to support this policy. Accordingly, mortgage lenders responded to the regulatory and market demands to make home buying accessible to low-income households through relaxing their mortgage underwriting standards such that households previously considered to be risky by traditional standards were able to access mortgage financing (Kiff and Mills, 2007).

The decline in mortgage underwriting standards was necessary since subprime borrowers are riskier by nature than prime borrowers. As such, even if the risk is accurately priced, providing mortgages to the low-income segment of the U.S. population required loosening underwriting standards. Listokin et al. (2000) note that even with the availability of perfect information for lenders, the financial characteristics of subprime households complicate the demand and supply sides of the subprime market. They state the following issues that make subprime households difficult bank customers:

– Insufficient Assets and/or Asset Verification: Low-income households are often unable to make required down payment, especially in rapidly appreciating markets. Low-pay employment and intermittent employment make it hard for these households to save for a down payment.

– Unsteady or Undocumented Income: Subprime households' employment nature makes it that they actually earn income but cannot prove it in the way most lenders require them to (Smith, 1998).

– Credit Problems: Subprime borrowers are usually unable to meet credit requirements set by lenders due to their uncertain repayment behaviour as reflected by their FICO[1] scores (Smith, 1998).

– Purchased Houses: The income, assets, and credit constraints faced by low-income households restrict their house purchasing options to low-priced neighbourhoods. As a

[1] The FICO (Fair Isaac Company) score rank-orders consumers by their likelihood of paying their credit obligations as agreed. The score range is [300, 850], with a higher score reflecting higher creditworthiness.

result, they purchase houses with higher perceived risk due to the uncertain prospects for house price appreciation in low-priced neighbourhoods (Listokin et al., 2000).

Given the above-mentioned issues, there is clearly no precise characterisation of subprime mortgage lending. As such, each underwriter independently evaluates the credit risk on the mortgage based on borrower-specific attributes like credit record and debt service-to-income ratio, as well as on the terms of the loan contract like the mortgage loan-to-value ratio (Kiff and Mills, 2007). Table 1 lists the main features of U.S. prime and subprime mortgage contracts during the years 2005 and 2006 as reported in Amromin and Paulson (2009). In the case of prime mortgages, more than 66% of the issued mortgages are conforming (i.e. conform to the guidelines of government sponsored enterprises – Freddie Mac and Fannie Mae) while only about 24% of subprime mortgages are conforming loans. Further, U.S. prime mortgages are usually collateralized and are characterised by a 30-year fixed interest rate, while borrowers with undocumented assets usually face an adjustable interest rate on subprime mortgage loans. As the down payment is usually higher in the case of prime mortgages, these loans are characterised by a lower LTV ratio (75% in 2005) than subprime mortgage loans (81% in 2005). Additionally, a major difference between the two types of loans lies in the FICO score that provides a measure of borrowers' creditworthiness. There is approximately 100 points difference in the FICO scores of U.S. prime and subprime borrowers implying that subprime mortgages are characterised by a higher risk of default.

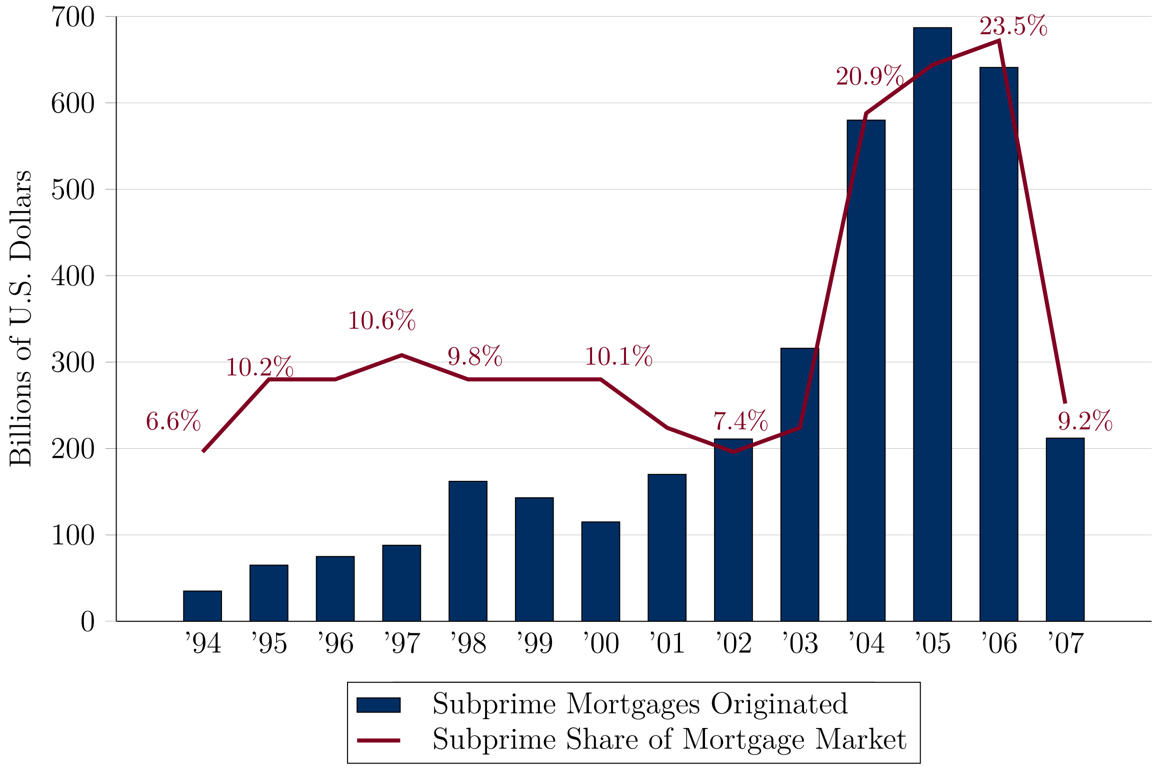
Table 1: U.S. Prime and Subprime Mortgages Characteristics

	Prime Mortgages		Subprime Mortgages	
	2005	2006	2005	2006
Origination Amount	\$200,383	\$211,052	\$172,316	\$179,003
FICO Score	715	708	611	607
Loan-to-Value Ratio	74.89%	75.99%	80.69%	80.40%
Debt-to-Income Ratio	37.87%	37.25%	38.35%	39.78%
Share of Adjustable Rate Mortgages	26.04%	23.16%	69.49%	61.78%
Share of Conforming Loans	66.50%	66.18%	24.47%	23.67%

Source: The data is retrieved from Amromin and Paulson (2009).
Notes: The characteristics of prime mortgages are based on a sample of 18,388 mortgages in 2005 and 15,992 mortgages in 2006. The characteristics of subprime mortgages are based on a sample of 20,778 mortgages in 2005 and 18,189 mortgages in 2006.

During the decade preceding the U.S. subprime mortgage crisis, the mortgage market has witnessed a quiet dramatic growth in subprime lending. The rise in subprime lending was driven by an overall boom in the U.S. housing market and was further facilitated by housing finance innovations. In fact, the seemingly persistent upward trend in house prices coupled with the growing demand for housing led mortgage lenders to ease underwriting standards on mortgage debt (higher loan-to-value ratios, less stringent debt service-to-income ratios, and limited required documentation on income and assets) in order to meet the rising demand for mortgage credit (Federal Reserve Bank of San Francisco, 2008). Consequently, home ownership rate for the U.S. increased from 64% in 1990 to a peak of 69% in 2004. Over the same period, the subprime mortgage market witnessed unprecedented growth, with subprime mortgage origination's share of total mortgage origination rising from 6.6% in 1994 to 23.5% in 2006 (See Figure 1).

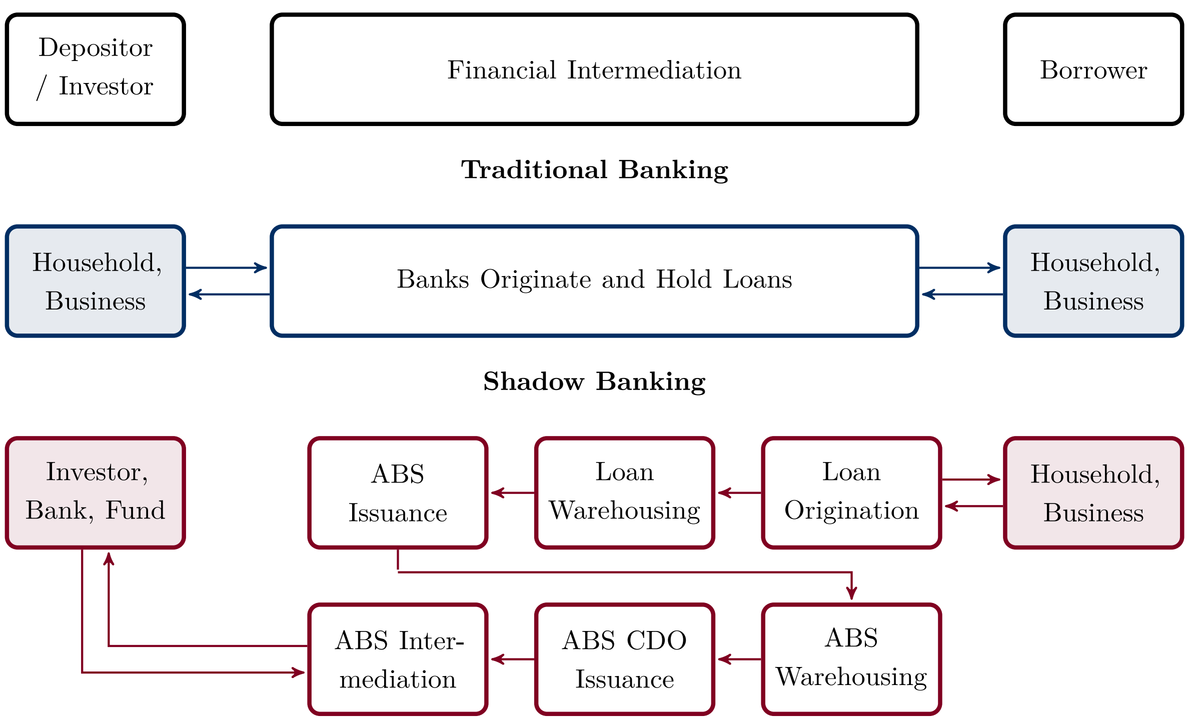
Figure 1: Subprime Mortgages Origination in the U.S.



Sources: Author's calculations; The Financial Crisis Inquiry Commission (2011); Inside Mortgage Finance.

The expansion of the subprime mortgage market was also facilitated by developments in financial intermediation channels. Traditional banking has long followed the originate-to-hold model in which banks issue loans to borrowers, hold these loans on their balance sheets, and finance the issuance through deposits from savers. Starting the 1970's, financial innovation and technological advancement led to a new avenue for financial intermediation that has been named shadow banking since it involves less transparency and regulation than traditional banking (Luttrell et al., 2012). Shadow banking follows the originate-to-distribute model in which lenders issue loans to borrowers with the intention of selling them to investors through a process known as securitization (See Figure 2). The process of securitisation involves pooling assets together and repackaging them into interest-paying securities. These asset-backed securities are usually pass-through securities in the sense that the principal and interest payments from the assets are passed to the investors in the securities (Jobst, 2008). Securitization started in the 1970's with prime residential mortgage loans being securitized by government-sponsored enterprises. But soon enough, securitization extended to other income-generating assets including corporate loans, consumer credit, and subprime mortgages.

Figure 2: Subprime Mortgages Origination in the U.S.



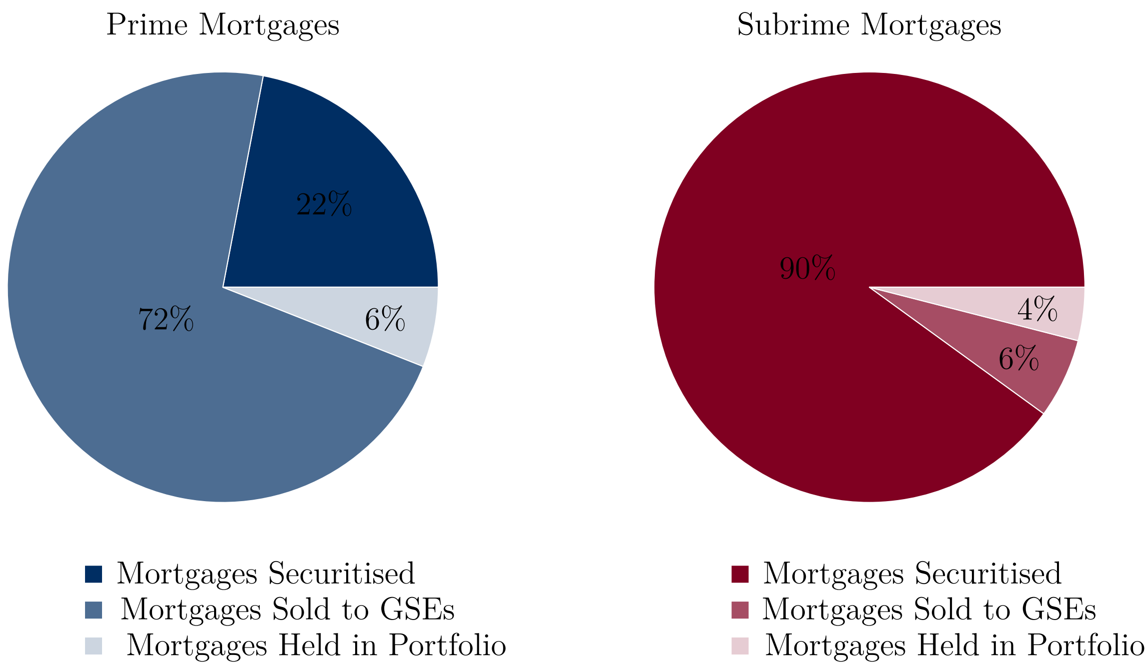
Source: Adapted from Luttrell et al. (2012).
Note: ABS is asset-backed securities and CDO is collateralized debt obligations.

In essence, securitization represents a source of financing based on credit risk transfer from the loan originator to investors. The process of securitization is initiated when the loans originator determines the assets to be securitized and pools them into a portfolio. The assets' pool is then sold to a special purpose vehicle (SPV) that is set up by the loan originator for the sole purpose of securitization. The SPV finances the acquisition of the assets' pool through issuing tradable interest-paying securities that are backed by these assets and selling them to investors. Investors in these derivative securities receive fixed or floating rate payments that are financed by the cash stream generated from the assets portfolio. The loans originator usually services the loans in the portfolio and collects the payments from the borrowers. The payments are then passed – net of a servicing fee paid to the originator – to the SPV (Jobst, 2008). It is noteworthy that the specific design of the SPV and the fact that it is set up by the loan originator allows me to incorporate the securitization process into the DSGE model without the need for introducing a new agent in the economy.

The derivative securities, also known as asset-backed securities, are further refined through dividing them into slices, called tranches. Each tranche has a different risk level associated to it and is sold independently. The varying risk levels stem from the fact that the investment returns and losses are allocated to the tranches based on their seniority. As such, the least risky tranche has the priority in receiving the income collected from the underlying loans while the riskiest tranche has the last call on the income. A typical asset-backed security has a three-tier design: senior, mezzanine, and equity tranches (Jobst, 2008). In this structure, the losses on the assets' portfolio are concentrated in the equity tranche that is usually the smallest tranche but the one that bears most of the risk. Tranching of the securitized products enables credit enhancement and meets the various risk appetites of investors. For instance, the senior tranche of asset-backed securities is usually purchased by commercial banks since it meets their demand for safe investments. Meanwhile, investors with an appetite for risky investments usually purchase the mezzanine and equity tranches.

For years, the government-sponsored enterprises have dominated the securitization of residential mortgages. The loans securitized by these enterprises have primarily been prime mortgages extended to high-quality borrowers. Yet, as the subprime market expanded and subprime mortgage originations grew, the originate-to-distribute model also came to dominate subprime financing (Federal Reserve Bank of San Francisco, 2008). In fact, the share of subprime mortgages financed through subprime mortgage-backed securities increased from 46% in 2001 to almost 80% by 2006 (Gorton, 2008). As such, securitization has constituted the main financing means for subprime mortgage originations prior to the U.S. subprime mortgage crisis (See Figure 3). Subprime mortgages were mostly pooled together and they backed the issuance of subprime mortgage-backed securities, which were often re-securitized into collateralized debt obligations (CDO). These subprime mortgage-backed securities, along with the assets they were packaged into, found their way into the portfolios of a wide range of investors including commercial banks, investment and pension funds, as well as personal investors. The securitization of subprime mortgages has exposed the financial system and the real economy to changes in housing fluctuations, as the honouring of the subprime mortgage obligations by borrowers and accordingly the value of the subprime mortgage-backed securities are directly associated with the developments in the housing market. Given the above, I solely focus on the securitization of subprime mortgages as the means for financing subprime originations in the model.

Figure 3: Subprime Mortgages Origination in the U.S.

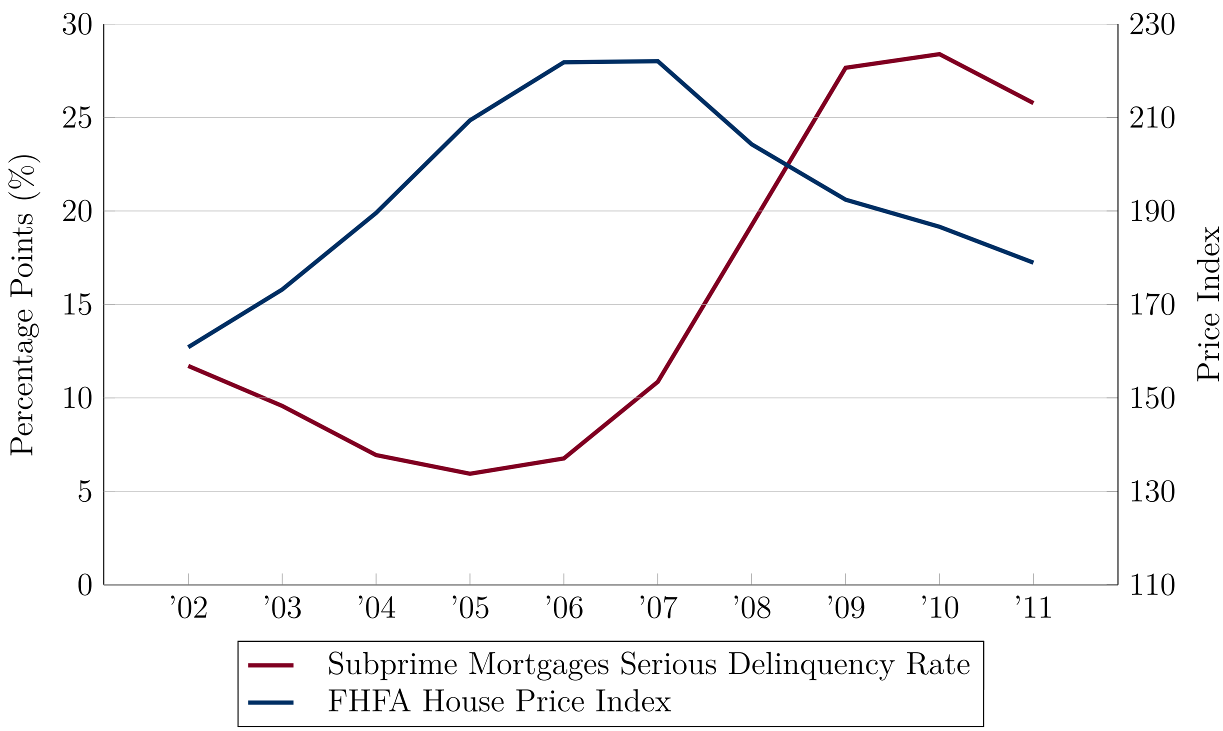


Source: The data is retrieved from Amromin and Paulson (2009).
Note: GSE is government-sponsored enterprise.

The sensitivity of the U.S. subprime mortgage market to the developments in the housing market became evident in the second half of 2006 and early 2007. With falling house prices and decreasing housing demand, the performance of subprime mortgage loans suddenly and substantially deteriorated (Federal Reserve Bank of San Francisco, 2008). In fact, as house prices growth turned negative, the burden on subprime mortgage debt became too heavy for borrowers who had no choice but to default on their loan obligations. Trouble in the subprime mortgage market started with early payment defaults (EPDs), which were soon enough followed by rising subprime delinquency and foreclosure rates. By the years 2007 and

2008, the U.S. subprime mortgage industry started collapsing with a growing number of home owners falling behind their mortgage payments and an unprecedented rise in foreclosure rates. As such, the subprime mortgage crisis revealed that house prices matter to the performance of subprime mortgages (See Figure 4) and several studies since then have shown that house prices constitute a major determinant of subprime mortgage default [2]. Accordingly, in the model I assume that the decision of subprime borrowers to honour their mortgage obligations is sensitive to house price changes. It is noteworthy that while mortgage defaults were initially concentrated in the subprime segment of the market, as the negative developments in the subprime market spilled-over to the real economy causing a recession, an increasing number of prime mortgages also became seriously delinquent (Wilse-Samson, 2010).

Figure 4: Subprime Mortgages Origination in the U.S.



Sources: Author's calculations; Mortgage Bankers Association; The Federal Housing Finance Agency.

The rise in subprime mortgage delinquencies and subprime foreclosures changed the market's assessment of the risk inherent in the subprime sector. Yet, despite the rise in delinquencies, the market appeared to still have confidence in highly rated senior tranches of subprime mortgage-backed securities and so the originate-to-distribute model continued to

[2] In a study by Gerardi et al. (2007), the authors conclude that house prices have been the main driver of the rising foreclosure rates during the U.S. subprime mortgage crisis.

function, though at a lower scale (Federal Reserve Bank of San Francisco, 2008). Nevertheless, as the risk indicators for subprime mortgage-backed securities began to rise in the second half of 2007, rating agencies downgraded the ratings of all the tranches of securities backed by subprime mortgages. In fact, when issues related to the repayment of securities backed by subprime mortgage surfaced in the riskiest tranches, it raised concerns about the more senior tranches and undermined investors' confidence in the securitized debt. Consequently, fire sale of the securitized debt prompted declines in the value of subprime mortgage-backed securities as market investors woke up to the fact that these securities were much less safe than they were thought to be.

Subsequently, investment banks found themselves stuck with billions of unsold securitized debt. Further, the banks and the investors in mortgage-backed securities and collateralised debt obligations foreclosed houses that were losing in value. Thus, as the demand for subprime mortgage-backed securities dried up, subprime mortgage lenders found themselves saddled with loans they had previously planned to securitize and were left holding nonconforming loans that they could not sell to the market. The result was a near seizing up of structured finance and a substantial cutback in subprime mortgage lending and securitization of subprime credit. Additionally, several financial institutions in the U.S. and abroad incurred sizeable losses due to their exposure as underwriters of structured debt, sponsors of special purpose vehicles, and investors in subprime-related securities. Also, many non-financial institutions and investors incurred losses on their investment in subprime mortgage-backed securities and thus experienced dramatic declines in their stock prices. At the same time, the financial difficulties spilled over to the real economy. With falling consumer and investor sentiment, the demand for both consumer and investment goods plummeted.

The fact that many investors, funds, as well as financial and non-financial institutions invested in securitized subprime debt means that everyone took losses. Thus, the scale and persistence of the U.S. subprime mortgage crisis suggests that securitization coupled with poor credit origination and loose underwriting standards, could severely hurt financial and economic stability (Jobst, 2008). This is why the subprime mortgage crisis represents, as Eichengreen (2008) puts it "the first crisis of the age of mass securitization". Hence, this paper attempts to analyse the mechanism by which securitization amplifies the response of the economy to housing shocks by exposing it to developments in the subprime mortgage market.

2 Other Elements of the Model

2.1 Capital Goods Producers

Firms that produce capital goods operate in a perfectly competitive environment. These firms buy undepreciated capital from the last period $(1-\delta)k_{t-1}$ at the price Q_t^k from entrepreneurs who are the owners of these firms. They also buy i_t units of final goods from retailers at the price P_t. Capital goods producers combine these inputs to produce the flow output given by $\Delta\bar{x}_t = k_t-(1-\delta)k_{t-1}$. The stock of capital $\bar{x}_t$ is sold back to entrepreneurs at the same price Q_t^k. Firms that produce capital goods maximise their expected real earnings:

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^E \left[q_t^k \Delta\bar{x}_t - i_t \right] \tag{1}$$

where $\Lambda_{0,t}^E \equiv \beta_E^t \lambda_t^E$ is the entrepreneur's relevant discount factor and $q_t^k \equiv Q_t^k / P_t$ is the real price of capital. When producing capital goods, capital goods producers face an adjustment cost due to transforming final goods into capital goods:

$$\bar{x}_t = \bar{x}_{t-1} + \left[1 - \frac{\kappa_i}{2} \left(\frac{i_t \varepsilon_t^{qk}}{i_{t-1}} - 1 \right)^2 \right] i_t \tag{2}$$

where κ_i determines the investment adjustment cost and ε_t^{qk} is a shock to investment efficiency.

Capital goods producers maximise their expected real earning, given in (1), subject to the investment adjustment cost, given in (2), with respect to the choice variables $\bar{x}_t$ and i_t. From the constraint, new capital is produced according to the law of motion:

$$k_t = (1-\delta)k_{t-1} + \left[1 - \frac{\kappa_i}{2} \left(\frac{i_t \varepsilon_t^{qk}}{i_{t-1}} - 1 \right)^2 \right] i_t. \tag{3}$$

From the first order condition, the real price of capital q_t^k is determined by:

$$1 = q_t^k \left[1 - \frac{\kappa_i}{2} \left(\frac{i_t \varepsilon_t^{qk}}{i_{t-1}} - 1 \right)^2 - \kappa_i \left(\frac{i_t \varepsilon_t^{qk}}{i_{t-1}} - 1 \right) \frac{i_t \varepsilon_t^{qk}}{i_{t-1}} \right] + \beta_E E_t \left[\frac{\lambda_{t+1}^E}{\lambda_t^E} q_{t+1}^k \varepsilon_{t+1}^{qk} \kappa_i \left(\frac{i_{t+1} \varepsilon_{t+1}^{qk}}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \right] \tag{4}$$

2.2 Final Goods Market

Each goods retailer purchases intermediate goods that are produced by the entrepreneurs at the price P_t^W then differentiates them into final goods $y_t(j)$ and sells them at the price

$P_t(j)$.

2.2.1 Final Goods Demand

The demand for differentiated final goods $y_t(j)$ is chosen to maximise the aggregation of these differentiated goods:

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon_t^y - 1}{\varepsilon_t^y}} dj\right]^{\frac{\varepsilon_t^y}{\varepsilon_t^y - 1}} \tag{5}$$

where ε_t^y is a stochastic parameter. This maximisation is subject to a budget constraint in which the spending on differentiated final goods shall not exceed an overall expenditure amount. The constraint is:

$$\int_0^1 P_t(j) y_t(j) dj \leq \bar{E}_t^y \tag{6}$$

where $P_t(j)$ is the nominal price set by the goods retailer j and $\bar{E}_t^y$ is the overall expenditure amount. The solution of this problem yields the demand for differentiated final goods:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_t^y} y_t \tag{7}$$

where $P_t \equiv \left[\int_0^1 P_t(j)^{1-\varepsilon_t^y} dj\right]^{\frac{1}{1-\varepsilon_t^y}}$ is the price index of retail goods.

2.2.2 Goods Retailers

Goods retailers operate in a monopolistically competitive environment and are a source of price stickiness in the economy. As in Gerali et al. (2010), the prices set by retailers are indexed to a weighted combination of previous period inflation π_{t-1} and steady-state inflation π, with weights given by ι_p and $1-\iota_p$, respectively . Hence, retailers face a quadratic adjustment cost if they change their prices beyond what is allowed by indexation. Goods retailers choose their price $P_t(j)$ in order to maximise their expected earnings:

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[P_t(j) y_t(j) - P_t^W y_t(j) - \frac{\kappa_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}\right)^2 P_t y_t\right] \tag{8}$$

where $\Lambda_{0,t}^P \equiv \beta_P^t \lambda_t^P$ is the patient household's relevant discount factor, and κ_p determines the cost for adjusting the price. The maximisation of expected earnings is subject to the demand for the final goods given in (7).

In a symmetric equilibrium, the first order condition yields the non-linear forward-looking

Phillips curve:

$$1-\varepsilon_t^y+\frac{\varepsilon_t^y}{x_t}-\kappa_p(\pi_t-\pi_{t-1}^{\iota_p}\pi^{1-\iota_p})\pi_t+\beta_p E_t\left[\frac{\lambda_{t+1}^P}{\lambda_t^P}\kappa_p\left(\pi_{t+1}-\pi_t^{\iota_p}\pi^{1-\iota_p}\right)\pi_{t+1}^2\frac{y_{t+1}}{y_t}\right]=0 \tag{9}$$

Also, aggregate goods retailers' profits in a symmetric equilibrium are given by:

$$\Omega_t^R=y_t\left[1-\frac{1}{x_t}-\frac{\kappa_p}{2}(\pi_t-\pi_{t-1}^{\iota_p}\pi^{1-\iota_p})^2\right] \tag{10}$$

2.3 Labour Market

Workers provide differentiated labour services. Unions sell these services to labour packers who assemble them as homogeneous labour and sell them to entrepreneurs.

2.3.1 Labour Packers

There are three perfectly competitive labour packers (indexed by s): patient, prime impatient, and subprime impatient labour packers. Furthermore, for each labour service of type $m \in [0,1]$, there are three types of unions (indexed also by s): for patient, prime impatient, and subprime impatient households. Each labour packer $s \in \{P, IP, IS\}$ demands differentiated labour services from unions (s,m) and assembles them into a constant elasticity of substitution (CES) composite labour input, which is in turn sold to entrepreneurs. The perfectly competitive labour packer chooses the demand for differentiated labour services $l_t^s(m)$ in order to maximise the aggregation of these services into homogeneous labour:

$$l_t^s=\left[\int_0^1 l_t^s(m)^{\frac{\varepsilon_t^l-1}{\varepsilon_t^l}}dm\right]^{\frac{\varepsilon_t^l}{\varepsilon_t^l-1}} \tag{11}$$

where ε_t^l is a stochastic parameter. This maximisation is subject to an expenditure constraint in which the spending on differentiated labour services shall not exceed an overall wage bill. The constraint is:

$$\int_0^1 W_t^s(m)l_t^s(m)dm \le \bar{E}_t^l \tag{12}$$

where $W_t^s(m)$ is the nominal wage set by the union (s,m) and $\bar{E}_t^l$ is the overall wage bill. The solution of the competitive labour packer's problem yields the demand for each type of differentiated labour services:

$$l_t^s(m)=\left(\frac{W_t^s(m)}{W_t^s}\right)^{-\varepsilon_t^l}l_t^s \tag{13}$$

where $W_t^s \equiv \left[\int_0^1 W_t^s(m)^{1-\varepsilon_t^l} dm\right]^{\frac{1}{1-\varepsilon_t^l}}$ is the aggregate wage index.

2.3.2 Unions

Workers (indexed by i) sell their differentiated labour through unions. Each union, denoted by (s, m), sets nominal wages $W_t^s(m)$ in order to maximise the utility of its members. Nominal wages are indexed to a weighted combination of previous period inflation π_{t-1} (weight ι_w) and steady-state inflation π (weight $1-\iota_w$). Thus, unions face an adjustment cost (with κ_w the adjustment cost parameter) if they change nominal wages beyond what indexation allows and charge member households a lump-sum fee to fund the adjustment cost. Unions choose $W_t^s(m)$ in order to maximise:

$$E_0 \sum_{t=0}^{\infty} \beta_s^t \left\{ U_{c_t^s(i,m)} \left[\frac{W_t^s(m)}{P_t} l_t^s(i,m) - \frac{\kappa_w}{2} \left(\frac{W_t^s(m)}{W_{t-1}^s(m)} - \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \right)^2 \frac{W_t^s}{P_t} \right] - \frac{l_t^s(i,m)^{1+\phi}}{1+\phi} \right\} \quad (14)$$

where $U_{c_t^s(i,m)}$ is the marginal utility of consumption of worker i with labour services of type m. The maximisation is subject to the demand from labour packers given in (13).

In a symmetric equilibrium, the first order condition yields the non-linear forward-looking wage-Phillips curve:

$$\kappa_w(\pi_t^{w^s} - \pi_{t-1}^{\iota_w} \pi^{1-\iota_w}) \pi_t^{w^s} = \beta_s E_t \left[\frac{\lambda_{t+1}^s}{\lambda_t^s} \kappa_w \left(\pi_{t+1}^{w^s} - \pi_t^{\iota_w} \pi^{1-\iota_w} \right) \frac{\pi_{t+1}^{w^s\,2}}{\pi_{t+1}} \right] + (1-\varepsilon_t^l) l_t^s + \frac{\varepsilon_t^l l_t^{s\,1+\phi}}{w_t^s \lambda_t^s} \quad (15)$$

where w_t^s is the real wage of household of type s and $\pi_t^{w^s} \equiv \frac{w_t^s}{w_{t-1}^s} \pi_t$ is the nominal type s wage inflation.

2.4 Loans, Deposits, and Securities Demand

I follow Gerali et al. (2010) in modeling market power in the banking industry using the framework in Dixit and Stiglitz (1977). Households, entrepreneurs, and commercial banks buy loans, deposit contracts, and mortgage-backed securities from a composite CES basket of differentiated financial products.

2.4.1 Prime Household Loan Demand

The representative prime impatient household i chooses the amount of real loans $b_t^{IP}(i,j)$ demanded from each commercial bank j in order to minimize the total interest repayment on the differentiated loans given by

$$\int_0^1 r_t^{bIP}(j) b_t^{IP}(i,j) dj \tag{16}$$

where $r_t^{bIP}(j)$ is the interest rate on prime household loans set by the commercial bank j. This minimization is subject to the condition that the amount of real loans $\bar{b}_t^{IP}(i)$ sought by the prime impatient household i is met. The condition is given by

$$\left[\int_0^1 b_t^{IP}(i,j)^{\frac{\varepsilon_t^{bIP}-1}{\varepsilon_t^{bIP}}} dj\right]^{\frac{\varepsilon_t^{bIP}}{\varepsilon_t^{bIP}-1}} \geq \bar{b}_t^{IP}(i) \tag{17}$$

where ε_t^{bIP} (> 1) is the interest rate elasticity of prime household loan demand. After aggregating over symmetric prime impatient households, the solution of this problem yields the demand for differentiated prime household loans

$$b_t^{IP}(j) = \left(\frac{r_t^{bIP}(j)}{r_t^{bIP}}\right)^{-\varepsilon_t^{bIP}} b_t^{IP} \tag{18}$$

where $r_t^{bIP} \equiv \left[\int_0^1 r_t^{bIP}(j)^{1-\varepsilon_t^{bIP}} dj\right]^{\frac{1}{1-\varepsilon_t^{bIP}}}$ is the interest rate index of prime household loans.

2.4.2 Subprime Household Loan Demand

Similar to prime impatient households, the representative subprime impatient household i chooses the amount of real loans $b_t^{IS}(i,k)$ demanded from each shadow bank k that would minimize the total repayment on the differentiated loans given by

$$E_t \int_0^1 \left[(1-\delta_{t+1}^{IS}) r_t^{bIS}(k) b_t^{IS}(i,k) - \delta_{t+1}^{IS} b_t^{IS}(i,k)\right] dk \tag{19}$$

where $r_t^{bIS}(k)$ is the interest rate on subprime household loans set by the shadow bank k. This minimization is subject to the constraint that the amount of real loans $\bar{b}_t^{IS}(i)$ sought by the subprime impatient household i is met. The constraint is given by

$$\left[\int_0^1 b_t^{IS}(i,k)^{\frac{\varepsilon_t^{bIS}-1}{\varepsilon_t^{bIS}}} dk\right]^{\frac{\varepsilon_t^{bIS}}{\varepsilon_t^{bIS}-1}} \geq \bar{b}_t^{IS}(i) \tag{20}$$

where ε_t^{bIS} (> 1) is the interest rate elasticity of subprime household loan demand. The solution of this problem, after aggregating over symmetric subprime impatient households, yields the demand for differentiated subprime household loans

$$b_t^{IS}(k) = E_t\left(\frac{(1-\delta_{t+1}^{IS})r_t^{bIS}(k) - \delta_{t+1}^{IS}}{(1-\delta_{t+1}^{IS})r_t^{bIS} - \delta_{t+1}^{IS}}\right)^{-\varepsilon_t^{bIS}} b_t^{IS} \tag{21}$$

where $E_t[(1-\delta_{t+1}^{IS})r_t^{bIS} - \delta_{t+1}^{IS}] \equiv E_t\left[\int_0^1 \left[(1-\delta_{t+1}^{IS})r_t^{bIS}(k) - \delta_{t+1}^{IS}\right]^{1-\varepsilon_t^{bIS}} dk\right]^{\frac{1}{1-\varepsilon_t^{bIS}}}$ is the net repayment index of subprime household loans.

2.4.3 Entrepreneur Loan Demand

The representative entrepreneur i demands differentiated real loans $b_t^E(i,j)$ from each commercial bank j. The decision on the amount of differentiated loans demanded is made in order to minimize the total interest repayment on these loans given by

$$\int_0^1 r_t^{bE}(j) b_t^E(i,j) dj \tag{22}$$

where $r_t^{bE}(j)$ is the interest rate on entrepreneur loans set by the commercial bank j. Entrepreneur i solves the above minimization problem subject to the condition that the total demand for real entrepreneur loans $\bar{b}_t^E(i)$ is met. This condition is given by

$$\left[\int_0^1 b_t^E(i,j)^{\frac{\varepsilon_t^{bE}-1}{\varepsilon_t^{bE}}} dj\right]^{\frac{\varepsilon_t^{bE}}{\varepsilon_t^{bE}-1}} \geq \bar{b}_t^E(i) \tag{23}$$

where ε_t^{bE} (> 1) is the interest rate elasticity of entrepreneur loan demand. The solution of this problem, after aggregating over symmetric entrepreneurs, yields the demand for differentiated entrepreneur loans

$$b_t^E(j) = \left(\frac{r_t^{bE}(j)}{r_t^{bE}}\right)^{-\varepsilon_t^{bE}} b_t^E \tag{24}$$

where $r_t^{bE} \equiv \left[\int_0^1 r_t^{bE}(j)^{1-\varepsilon_t^{bE}} dj\right]^{\frac{1}{1-\varepsilon_t^{bE}}}$ is the interest rate index of entrepreneur loans.

2.4.4 Deposit Demand

The demand for differentiated deposit contracts $d_t^P(i,j)$ by the representative patient household i from each commercial bank j is obtained by maximizing the total interest revenue on these deposits given by

$$\int_0^1 r_t^d(j) d_t^P(i,j) dj \tag{25}$$

where $r_t^d(j)$ is the interest rate on deposits set by the commercial bank j. The decision on the differentiated deposit contracts is subject to the condition that the total amount of differentiated deposit contracts demanded does not exceed the overall amount of real savings $\bar{d}_t^P(i)$ of patient household i. This condition is given by

$$\left[\int_0^1 d_t^P(i,j)^{\frac{\varepsilon_t^d-1}{\varepsilon_t^d}} dj\right]^{\frac{\varepsilon_t^d}{\varepsilon_t^d-1}} \leq \bar{d}_t^P(i) \tag{26}$$

where ε_t^d (< -1) is the interest rate elasticity of deposit demand. After aggregating over symmetric patient households, the solution of this problem yields the demand for differentiated deposit contracts

$$d_t^P(j) = \left(\frac{r_t^d(j)}{r_t^d}\right)^{-\varepsilon_t^d} d_t^P \tag{27}$$

where $r_t^d \equiv \left[\int_0^1 r_t^d(j)^{1-\varepsilon_t^d} dj\right]^{\frac{1}{1-\varepsilon_t^d}}$ is the interest rate index of deposit contracts.

2.4.5 Mortgage-Backed Securities' Senior Tranche Demand

Demand by commercial bank j seeking an amount of the senior tranche of the mortgage-backed securities equal to $\bar{m}_t^s(j)$ is derived from maximizing over $m_t^s(j,k)$ the total return received from the continuum of shadow banks k that is given by

$$\int_0^1 \left[(1+r_t^{bIS}(k))\frac{F_t^s}{1-f} - p_t^s(k)\right] m_t^s(j,k) dk \tag{28}$$

where $p_t^s(k)$ is the price of the senior tranche of the mortgage-backed securities set by the shadow bank k. This maximization is subject to the constraint

$$\left[\int_0^1 m_t^s(j,k)^{\frac{\varepsilon_t^{ms}-1}{\varepsilon_t^{ms}}} dk\right]^{\frac{\varepsilon_t^{ms}}{\varepsilon_t^{ms}-1}} \leq \bar{m}_t^s(j) \tag{29}$$

where ε_t^{ms} (< -1) is the price elasticity of demand for the senior tranche of mortgage-backed securities. The solution of this problem, after aggregating over symmetric commercial banks, yields the demand for differentiated senior mortgage-backed securities

$$m_t^s(k) = \left(\frac{(1+r_t^{bIS}(k))\frac{F_t^s}{1-f} - p_t^s(k)}{(1+r_t^{bIS})\frac{F_t^s}{1-f} - p_t^s}\right)^{-\varepsilon_t^{ms}} m_t^s \tag{30}$$

where $(1+r_t^{bIS})\frac{F_t^s}{1-f} - p_t^s \equiv \left[\int_0^1 [(1+r_t^{bIS}(k))\frac{F_t^s}{1-f} - p_t^s(k)]dk\right]^{\frac{1}{1-\varepsilon_t^{ms}}}$ is the return index of the senior tranche of mortgage-backed securities.

2.4.6 Mortgage-Backed Securities' Equity Tranche Demand

The demand for differentiated equity tranches of mortgage-backed securities $m_t^e(i,k)$ by the representative entrepreneur i from each shadow bank k is obtained by maximizing the total return on these securities given by

$$\int_0^1 \left[(1+r_t^{bIS}(k))\frac{F_t^e}{f} - p_t^e(k)\right] m_t^e(i,k)dk \tag{31}$$

where $p_t^e(k)$ is the price of the equity tranche of the mortgage-backed securities set by the shadow bank k. This decision is subject to the condition that the aggregate demand for differentiated equity mortgage-backed securities does not exceed the total amount of investment $\bar{m}_t^e(i)$ by entrepreneur i. The condition is given by

$$\left[\int_0^1 m_t^e(i,k)^{\frac{\varepsilon_t^{me}-1}{\varepsilon_t^{me}}} dk\right]^{\frac{\varepsilon_t^{me}}{\varepsilon_t^{me}-1}} \leq \bar{m}_t^e(i) \tag{32}$$

where ε_t^{me} (< -1) is the price elasticity of demand for the equity tranche of mortgage-backed securities. The solution of this problem, after aggregating over symmetric entrepreneurs, yields the demand for differentiated equity mortgage-backed securities

$$m_t^e(k) = \left(\frac{(1+r_t^{bIS}(k))\frac{F_e^s}{f} - p_t^e(k)}{(1+r_t^{bIS})\frac{F_t^e}{f} - p_t^e}\right)^{-\varepsilon_t^{me}} m_t^e \tag{33}$$

where $(1+r_t^{bIS})\frac{F_t^e}{f} - p_t^e \equiv \left[\int_0^1 [(1+r_t^{bIS}(k))\frac{F_t^e}{f} - p_t^e(k)]dk\right]^{\frac{1}{1-\varepsilon_t^{me}}}$ is the return index of the equity tranche of mortgage-backed securities.

3 Steady State of the Model

3.1 Patient Households in Steady State

$$\frac{\varepsilon^c}{c^P} = \lambda^P \tag{34}$$

$$\frac{\varepsilon^h \gamma_h}{h^P} = \lambda^P q^h (1 - \beta_P) \tag{35}$$

$$\beta_P (1 + r^d) = \pi \tag{36}$$

$$c^P + d^P = w^P l^P + \frac{(1 + r^d)}{\pi} d^P + \Omega^R + \omega_b \frac{\Omega_b}{\pi} + \omega_s \frac{\Omega_s}{\pi} \tag{37}$$

3.2 Prime Impatient Households in Steady State

$$\frac{\varepsilon^c}{c^{IP}} = \lambda^{IP} \tag{38}$$

$$\frac{\varepsilon^h \gamma_h}{h^{IP}} + s^{IP} f^{IP} q^h \pi = \lambda^{IP} q^h (1 - \beta_{IP}) \tag{39}$$

$$s^{IP} (1 + r^{bIP}) = \lambda^{IP} \left(1 - \beta_{IP} \frac{(1 + r^{bIP})}{\pi} \right) \tag{40}$$

$$c^{IP} + \frac{(1 + r^{bIP})}{\pi} b^{IP} = w^{IP} l^{IP} + b^{IP} \tag{41}$$

$$(1 + r^{bIP}) b^{IP} = f^{IP} q^h h^{IP} \pi \tag{42}$$

3.3 Subprime Impatient Households in Steady State

$$\frac{\varepsilon^c}{c^{IS}} = \lambda^{IS} \tag{43}$$

$$\frac{\varepsilon^h \gamma_h}{h^{IS}} + s^{IS} f^{IS} q^h (1 - \delta^{IS}) \pi = \lambda^{IS} q^h (1 - \beta_{IS}) \tag{44}$$

$$s^{IS}(1+r^{bIS}) = \lambda^{IS}\left(1-\beta_{IS}\frac{(1+r^{bIS})(1-\delta^{IS})}{\pi}\right) \tag{45}$$

$$c^{IS} + \frac{(1+r^{bIS})(1-\delta^{IS})}{\pi}b^{IS} = w^{IS}l^{IS} + b^{IS} \tag{46}$$

$$(1+r^{bIS})b^{IS} = f^{IS}q^h h^{IS}(1-\delta^{IS})\pi \tag{47}$$

3.4 Entrepreneurs in Steady State

$$\frac{1}{c^E} = \lambda^E \tag{48}$$

$$s^E f^E q^k \pi(1-\delta) + \beta_E \lambda^E\Big(q^k(1-\delta) + r^k u\Big) = \lambda^E q^k \tag{49}$$

$$s^E(1+r^{bE}) = \lambda^E\left(1-\beta_E\frac{(1+r^{bE})}{\pi}\right) \tag{50}$$

$$fp^e = \beta_E\frac{F^e(1+r^{bIS})}{\pi} \tag{51}$$

$$w^P = \frac{(1-\alpha)\mu_1 y^E}{xl^{E,P}} \tag{52}$$

$$w^{IP} = \frac{(1-\alpha)\mu_2 y^E}{xl^{E,IP}} \tag{53}$$

$$w^{IS} = \frac{(1-\alpha)(1-\mu_1-\mu_2)y^E}{xl^{E,IS}} \tag{54}$$

$$r^k = \xi_1 \tag{55}$$

$$r^k = \frac{\alpha a^E\Big[k^E u\Big]^{\alpha-1}\Big[(l^{E,P})^{\mu_1}(l^{E,IP})^{\mu_2}(l^{E,IS})^{1-\mu_1-\mu_2}\Big]^{1-\alpha}}{x} \tag{56}$$

$$c^E + w^P l^{E,P} + w^{IP}l^{E,IP} + w^{IS}l^{E,IS} + \frac{(1+r^{bE})}{\pi}b^E + p^e m^e = \frac{y^E}{x} + b^E + \frac{F^e(1+r^{bIS})m^e}{f\pi} - \delta q^k k^E \tag{57}$$

$$(1 + r^{bE})b^E = f^E q^k (1-\delta) k^E \pi \tag{58}$$

$$y^E = a^E \Big[k^E u\Big]^{\alpha} \Big[(l^{E,P})^{\mu_1} (l^{E,IP})^{\mu_2} (l^{E,IS})^{1-\mu_1-\mu_2}\Big]^{1-\alpha} \tag{59}$$

3.5 Capital Goods Producers in Steady State

$$k - (1-\delta)k = i \tag{60}$$

$$q^k = 1 \tag{61}$$

3.6 Final Goods Market in Steady State

$$x = \frac{\varepsilon^y}{\varepsilon^y - 1} \tag{62}$$

$$\Omega^R = y\left(1 - \frac{1}{x}\right) \tag{63}$$

3.7 Labour Market in Steady State

$$l^{P\phi} = \frac{\varepsilon^l - 1}{\varepsilon^l} w^P \lambda^P \tag{64}$$

$$l^{IP\phi} = \frac{\varepsilon^l - 1}{\varepsilon^l} w^{IP} \lambda^{IP} \tag{65}$$

$$l^{IS\phi} = \frac{\varepsilon^l - 1}{\varepsilon^l} w^{IS} \lambda^{IS} \tag{66}$$

3.8 Commercial Banks in Steady State

$$\pi K^b = (1-\delta^b) K^b + (1-\omega_b)\Omega^b \tag{67}$$

$$B + p^s m^s = D + K^b \tag{68}$$

$$R^b = R^d \tag{69}$$

$$\frac{F^s(1+r^{bIS})}{(1-f)p^s} - 1 = R^d \tag{70}$$

$$R^d = r \tag{71}$$

$$B = b^{IP} + b^E \tag{72}$$

$$D = d^P \tag{73}$$

$$r^{bIP} = \frac{\varepsilon^{bIP}}{\varepsilon^{bIP} - 1} R^b \tag{74}$$

$$r^{bE} = \frac{\varepsilon^{bE}}{\varepsilon^{bE} - 1} R^b \tag{75}$$

$$r^d = \frac{\varepsilon^d}{\varepsilon^d - 1} R^d \tag{76}$$

$$\Omega^b = r^{bIP} b^{IP} + r^{bE} b^E - r^d d^P + \frac{F^s(1+r^{bIS})}{1-f} m^s - p^s m^s \tag{77}$$

3.9 Shadow Banks in Steady State

$$\pi K^s = (1-\delta^s) K^s + (1-\omega_s)\Omega^s \tag{78}$$

$$B^{IS} = P^s M^s + P^e M^e + K^s \tag{79}$$

$$(1-f)P^s + f P^e = 1 \tag{80}$$

$$B^{IS} = b^{IS} \tag{81}$$

$$M^s = m^s \tag{82}$$

$$M^e = m^e \tag{83}$$

$$(1-\delta^{IS})r^{bIS} - \delta^{IS} = \frac{\varepsilon^{bIS}}{\varepsilon^{bIS}-1}R^{bIS} \tag{84}$$

$$(1+r^{bIS})\frac{F^s}{1-f} - p^s = \frac{\varepsilon^{ms}}{\varepsilon^{ms}-1}\Big((1+R^{bIS}) - P^s\Big) \tag{85}$$

$$(1+r^{bIS})\frac{F^e}{f} - p^e = \frac{\varepsilon^{me}}{\varepsilon^{me}-1}\Big((1+R^{bIS}) - P^e\Big) \tag{86}$$

$$\Omega^s = \Big(r^{bIS}(1-\delta^{IS}) - \delta^{IS}\Big)b^{IS} + p^s m^s + p^e m^e - (1+r^{bIS})\left(\frac{F^s}{1-f}m^s + \frac{F^e}{f}m^e\right) \tag{87}$$

3.10 Market Clearing in Steady State

$$y = c + \Big(k - (1-\delta)k\Big) + \delta^b \frac{K^b}{\pi} + \delta^s \frac{K^s}{\pi} \tag{88}$$

$$c = c^P + c^{IP} + c^{IS} + c^E \tag{89}$$

$$\bar{h} = h^P + h^{IP} + h^{IS} \tag{90}$$

$$y = y^E \tag{91}$$

$$k = k^E \tag{92}$$

$$l^P = l^{E,P} \tag{93}$$

$$l^{IP} = l^{E,IP} \tag{94}$$

$$l^{IS} = l^{E,IS} \tag{95}$$

References

Amromin, G. and Paulson, A. L. (2009). Comparing Patterns of Default among Prime and Subprime Mortgages. *Economic Perspectives*, 33(2):18–37.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *The American Economic Review*, 67(3):297–308.

Eichengreen, B. (2008). Ten Questions About the Subprime Crisis. Banque de France, Financial Stability Review No. 11.

Federal Reserve Bank of San Francisco (2008). The Subprime Mortgage Market: National and Twelfth District Developments. Annual Report 2007.

Gerali, A., Neri, S., Sessa, L., and Signoretti, F. M. (2010). Credit and Banking in a DSGE Model of the Euro Area. *Journal of Money, Credit and Banking*, 42(6):107–141.

Gerardi, K., Shapiro, A., and Willen, P. (2007). Subprime Outcomes: Risky Mortgages, Homeownership Experiences, and Foreclosures. Federal Reserve Bank of Boston, Working Paper 07-15.

Gorton, G. B. (2008). The Panic of 2007. NBER Working Paper No. 14358.

Jobst, A. (2008). What Is Securitization? IMF Monetary and Capital Markets Department.

Kiff, J. and Mills, P. (2007). Money for Nothing and Checks for Free: Recent Developments in U.S. Subprime Mortgage Markets. IMF Working Paper WP/07/188.

Listokin, D., Wyly, E. K., Keating, L., Rengert, K. M., and Listokin, B. (2000). Making New Mortgage Markets: Case Studies of Institutions, Home Buyers, and Communities. Fannie Mae Foundation.

Luttrell, D., Rosenblum, H., and Thies, J. (2012). Understanding the Risks Inherent in Shadow Banking: A Primer and Practical Lessons Learned. Staff Papers 18, Federal Reserve Bank of Dallas.

Smith, S. (1998). Interview with National Manager of Community Lending, Bank of America. Interview by David Listokin and Barbara Listokin, Center for Urban Policy Research, Rutgers University.

The Financial Crisis Inquiry Commission (2011). The Financial Crisis Inquiry Report. Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States.

Wilse-Samson, L. (2010). The Subprime Mortgage Crisis: Underwriting Standards, Loan Modifications and Securitization. Working Paper.