Fiscal Policy and the Slowdown in Trend Growth in an Open Economy

Online Appendix

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1 Model Equations

1.1 Non-Stationary Equations

$$\Lambda_t(1+\tau_t^c) = \frac{\zeta_t}{(C_t - hC_{t-1})^\sigma} - \beta h \mathbb{E}_t \left\{ \frac{\zeta_{t+1}}{(C_{t+1} - hC_t)^\sigma} \right\} \tag{1}$$

$$\Lambda_t = \beta R_t \mathbb{E}_t \Lambda_{t+1} \tag{2}$$

$$\Lambda_t = \beta R_t^F \mathbb{E}_t \Lambda_{t+1} \tag{3}$$

$$\Phi_t = \beta \mathbb{E}_t \ \left[\Lambda_{t+1}(1-\tau_{t+1}^K) r_{t+1}^K + \Phi_{t+1}(1-\delta)\right] \tag{4}$$

$$\Lambda_t = \Phi_t \zeta_t^I \left[1 - \Upsilon\left(\frac{I_t}{I_{t-1}}\right) - \Upsilon'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \beta \mathbb{E}_t \ \Phi_{t+1}\zeta_{t+1}^I \Upsilon'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}^2}{I_t^2} \tag{5}$$

$$\gamma \zeta_t \zeta_t^L Z_t^{1-\sigma} L_t^\nu = \Lambda_t (1-\tau_t^w) W_t \tag{6}$$

$$K_t = (1-\delta) K_{t-1} + \zeta_t^I \left[1 - \Upsilon\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{7}$$

$$Y_t = K_{t-1}^\alpha (Z_t L_t)^{1-\alpha} \tag{8}$$

$$r_t^K = \alpha \frac{Y_t}{K_{t-1}} \tag{9}$$

$$W_t = (1-\alpha)\frac{Y_t}{L_t} \tag{10}$$

$$NX_t = Y_t - C_t - I_t - G_t \tag{11}$$

$$CA_t = NX_t + (R_{t-1}^F - 1) B_{t-1}^F \tag{12}$$

$$B_t^F = R_{t-1}^F B_{t-1}^F + NX_t \tag{13}$$

$$R_t^F = R_t^* \exp\left[-\psi_b\left(\frac{b_t^F}{y_t} - \frac{b^F}{y}\right) + \zeta_t^b\right] \tag{14}$$

$$G_t = B_t - R_{t-1}B_{t-1} + \tau_t^c C_t + \tau_t^w W_t L_t + \tau_t^K r_t^K K_{t-1} + TR_t \tag{15}$$

$$\ln z_t = (1-\rho_z)\ln z + \rho_z \ln z_{t-1} + \varepsilon_{z,t} \tag{16}$$

$$\ln R_t^* = (1-\rho_{R^*})\ln R^* + \rho_{R^*}\ln R_{t-1}^* + \varepsilon_{R^*,t} \tag{17}$$

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} - (1-\rho_g)\gamma_{gb}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{g,t} \tag{18}$$

$$\tau_t^c = (1-\rho_c)\tau_c + \rho_c \tau_{t-1}^c + (1-\rho_c)\gamma_{cb}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{c,t} \tag{19}$$

$$\tau_t^w = (1-\rho_w)\tau_w + \rho_w \tau_{t-1}^w + (1-\rho_w)\gamma_{wb}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{w,t} \tag{20}$$

$$\tau_t^K = (1-\rho_K)\tau_K + \rho_K \tau_{t-1}^K + (1-\rho_K)\gamma_{Kb}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{K,t} \tag{21}$$

$$\tau_t = (1-\rho_\tau)\tau + \rho_\tau \tau_{t-1} + (1-\rho_\tau)\gamma_{\tau b}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{\tau,t} \tag{22}$$

$$\ln \zeta_t = \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta,t} \tag{23}$$

$$\ln \zeta_t^L = \rho_L \ln \zeta_{t-1}^L + \varepsilon_{L,t} \tag{24}$$

$$\ln \zeta_t^I = \rho_I \ln \zeta_{t-1}^I + \varepsilon_{I,t} \tag{25}$$

$$\zeta_t^b = (1-\rho_b)\zeta^b + \rho_b \zeta_{t-1}^b + \varepsilon_{b,t} \tag{26}$$

1.2 Stationary Equations

The normalised variables are as follows:

1. $c_t = \frac{C_t}{Z_t}$
2. $\lambda_t = \Lambda_t Z_t^\sigma$
3. $\phi_t = \Phi_t Z_t^\sigma$
4. $i_t = \frac{I_t}{Z_t}$
5. $k_t = \frac{K_t}{Z_t}$
6. $l_t = L_t$
7. $r_t^F = R_t^F$
8. $r_t = R_t$
9. $w_t = \frac{W_t}{Z_t}$
10. $y_t = \frac{Y_t}{Z_t}$
11. $b_t^F = \frac{B_t^F}{Z_t}$
12. $b_t = \frac{B_t}{Z_t}$
13. $nx_t = \frac{NX_t}{Z_t}$
14. $ca_t = \frac{CA_t}{Z_t}$
15. $r_t^* = R_t^*$
16. $g_t = \frac{G_t}{Z_t}$
17. $\tau_t = \frac{TR_t}{Z_t}$

$$\lambda_t(1+\tau_t^c) = \frac{\zeta_t z_t^\sigma}{(c_t z_t - h c_{t-1})^\sigma} - \beta h \mathbb{E}_t\left\{\frac{\zeta_{t+1}}{(c_{t+1} z_{t+1} - h c_t)^\sigma}\right\} \tag{27}$$

$$\lambda_t = \beta R_t \mathbb{E}_t\left(\frac{\lambda_{t+1}}{z_{t+1}^\sigma}\right) \tag{28}$$

$$\lambda_t = \beta R_t^F \mathbb{E}_t\left(\frac{\lambda_{t+1}}{z_{t+1}^\sigma}\right) \tag{29}$$

$$\phi_t = \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}}{z_{t+1}^\sigma}(1-\tau_{t+1}^K) r_{t+1}^K + \frac{\phi_{t+1}}{z_{t+1}^\sigma}(1-\delta)\right] \tag{30}$$

$$\lambda_t = \phi_t \zeta_t^I \left[1 - \Upsilon\left(\frac{i_t z_t}{i_{t-1}}\right) - \Upsilon'\left(\frac{i_t z_t}{i_{t-1}}\right)\frac{i_t z_t}{i_{t-1}}\right] + \beta \mathbb{E}_t \frac{\phi_{t+1}\zeta_{t+1}^I}{z_{t+1}^\sigma}\Upsilon'\left(\frac{i_{t+1} z_{t+1}}{i_t}\right)\left(\frac{i_{t+1} z_{t+1}}{i_t}\right)^2 \tag{31}$$

$$\gamma\zeta_t\zeta_t^L l_t^\nu = \lambda_t(1-\tau_t^w)w_t \tag{32}$$

$$k_t = (1-\delta)\,\frac{k_{t-1}}{z_t} + \zeta_t^I\left[1-\Upsilon\left(\frac{i_t z_t}{i_{t-1}}\right)\right] i_t \tag{33}$$

$$y_t = k_{t-1}^\alpha l_t^{1-\alpha} z_t^{-\alpha} \tag{34}$$

$$r_t^K = \alpha\frac{y_t z_t}{k_{t-1}} \tag{35}$$

$$w_t = \frac{(1-\alpha)y_t}{l_t} \tag{36}$$

$$nx_t = y_t - c_t - i_t - g_t \tag{37}$$

$$ca_t = nx_t + (r_{t-1}^F - 1)\frac{b_{t-1}^F}{z_t} \tag{38}$$

$$b_t^F = \frac{r_{t-1}^F}{z_t} b_{t-1}^F + nx_t \tag{39}$$

$$r_t^F = r_t^* \exp\left[-\psi_b\left(\frac{b_t^F}{y_t} - \frac{b^F}{y}\right) + \zeta_t^b\right] \tag{40}$$

$$g_t + \frac{r_{t-1}b_{t-1}}{z_t} = b_t + \tau_t^c c_t + \tau_t^w w_t l_t + \frac{\tau_t^K r_t^K k_{t-1}}{z_t} + \tau_t \tag{41}$$

$$\ln z_t = (1-\rho_z)\ln z + \rho_z \ln z_{t-1} + \varepsilon_{z,t} \tag{42}$$

$$\ln r_t^* = (1-\rho_{r^*})\ln r^* + \rho_{r^*}\ln r_{t-1}^* + \varepsilon_{r^*,t} \tag{43}$$

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} - (1-\rho_g)\gamma_{gb}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{g,t} \tag{44}$$

$$\tau_t^c = (1-\rho_c)\tau_c + \rho_c\tau_{t-1}^c + (1-\rho_c)\gamma_{cb}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{c,t} \tag{45}$$

$$\tau_t^w = (1-\rho_w)\tau_w + \rho_w\tau_{t-1}^w + (1-\rho_w)\gamma_{wb}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{w,t} \tag{46}$$

$$\tau_t^K = (1-\rho_K)\tau_K + \rho_K\tau_{t-1}^K + (1-\rho_K)\gamma_{Kb}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{K,t} \tag{47}$$

$$\tau_t = (1-\rho_\tau)\tau + \rho_\tau\tau_{t-1} + (1-\rho_\tau)\gamma_{\tau b}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) + \varepsilon_{\tau,t} \tag{48}$$

$$\ln\zeta_t = \rho_\zeta \ln\zeta_{t-1} + \varepsilon_{\zeta,t} \tag{49}$$

$$\ln\zeta_t^L = \rho_L \ln\zeta_{t-1}^L + \varepsilon_{L,t} \tag{50}$$

$$\ln\zeta_t^I = \rho_I \ln\zeta_{t-1}^I + \varepsilon_{I,t} \tag{51}$$

$$\zeta_t^b = (1-\rho_b)\zeta^b + \rho_b\zeta_{t-1}^b + \varepsilon_{b,t} \tag{52}$$

1.3 Steady State

$$\frac{z^\sigma - h\beta}{c^\sigma (z-h)^\sigma} = \lambda(1+\tau^c) \tag{53}$$

$$r^F = \frac{z^\sigma}{\beta} \tag{54}$$

$$r = \frac{z^\sigma}{\beta} \tag{55}$$

$$(1-\tau^K) r^K = \frac{z^\sigma}{\beta} + \delta - 1 \tag{56}$$

$$\lambda = \phi \tag{57}$$

$$l^\nu = \frac{\lambda(1-\tau^w) w}{\gamma} \tag{58}$$

$$i = k - (1-\delta)\,\frac{k}{z} \tag{59}$$

$$y = k^\alpha\,(l)^{1-\alpha}\,z^{-\alpha} \tag{60}$$

$$r^K = \alpha \frac{yz}{k} \tag{61}$$

$$w = \frac{(1-\alpha)y}{l} \tag{62}$$

$$nx = y - c - i - g \tag{63}$$

$$ca = nx + (r^F - 1)\frac{b^F}{z} \tag{64}$$

$$b^F = \frac{r^F}{z} b^F + nx \tag{65}$$

$$r^F = r^* \exp(\zeta^b) \tag{66}$$

$$g + (\frac{1}{\beta} - 1) b = \tau^c c + \tau^w w l + \frac{\tau^K r^K k}{z} + \tau \tag{67}$$

1.4 Log-Linear Equations and Observation Equations

Tax rates, debt, foreign debt, the current account and trade balance are defined in terms of deviations from their steady state values. For these variables we use the notation, $\tilde{x}_t = x_t - x$. Remaining variables are expressed in log-deviations, that is $\hat{x}_t = \ln x_t - \ln x$.

1.4.1 Structural Equations

$$\begin{aligned} 0 = {} & (z^\sigma - h\beta)(z-h)\left(\hat{\lambda}_t + \frac{1}{1+\tau^c}\tilde{\tau}^c_t\right) + (\sigma z^{\sigma+1} + \sigma h^2 \beta)\hat{c}_t - h\beta\sigma z \hat{c}_{t+1} - h\sigma z^\sigma \hat{c}_{t-1} + \sigma h z^\sigma \hat{z}_t \\ & - h\beta\sigma z \hat{z}_{t+1} - (z-h)(z^\sigma \hat{\zeta}_t - h\beta\hat{\zeta}_{t+1}) \end{aligned} \tag{68}$$

$$0 = -\hat{\lambda}_t + \hat{\lambda}_{t+1} + \hat{r}_t - \sigma \hat{z}_{t+1} \tag{69}$$

$$0 = -\hat{\lambda}_t + \hat{\lambda}_{t+1} + \hat{r}_t^F - \sigma \hat{z}_{t+1} \tag{70}$$

$$0 = -\hat{\phi}_t + \frac{\beta(1-\tau^K)r^K}{z^\sigma}\left(\hat{\lambda}_{t+1} - \sigma\hat{z}_{t+1} + \hat{r}^K_{t+1}\right) - \frac{\beta r^K}{z^\sigma}\tilde{\tau}^K_{t+1} + \frac{\beta(1-\delta)}{z^\sigma}\left(\hat{\phi}_{t+1} - \sigma\hat{z}_{t+1}\right) \tag{71}$$

$$0 = -\hat{\lambda}_t + \hat{\phi}_t + \hat{\zeta}^I_t - z^2\Upsilon''\left((1+\beta z^{1-\sigma})\hat{i}_t - \hat{i}_{t-1} - \beta z^{1-\sigma}\hat{i}_{t+1} + \hat{z}_t - \beta z^{1-\sigma}\hat{z}_{t+1}\right) \tag{72}$$

$$0 = \hat{\lambda}_t + \hat{w}_t - \frac{1}{1-\tau^w}\tilde{\tau}^w_t - \nu\hat{l}_t - \hat{\zeta}^L_t - \hat{\zeta}_t \tag{73}$$

$$0 = \hat{k}_t - \frac{1-\delta}{z}(\hat{k}_{t-1} - \hat{z}_t) - \frac{z-1+\delta}{z}(\hat{i}_t + \hat{\zeta}^I_t) \tag{74}$$

$$0 = \hat{y}_t - \alpha\hat{k}_{t-1} - (1-\alpha)\hat{l}_t + \alpha\hat{z}_t \tag{75}$$

$$0 = \hat{y}_t + \hat{z}_t - \hat{k}_{t-1} - \hat{r}^K_t \tag{76}$$

$$0 = \hat{y}_t - \hat{l}_t - \hat{w}_t \tag{77}$$

$$0 = -n\tilde{x}_t + y\hat{y}_t - c\hat{c}_t - i\hat{i}_t - g\hat{g}_t \tag{78}$$

$$0 = -c\tilde{a}_t + n\tilde{x}_t + \frac{r^F b^F}{z}\hat{r}^F_{t-1} + \left(\frac{r^F-1}{z}\right)\tilde{b}^F_{t-1} - \left(\frac{r^F-1}{z}\right)b^F\hat{z}_t \tag{79}$$

$$0 = -\tilde{b}^F_t + n\tilde{x}_t + \frac{r^F}{z}\tilde{b}^F_{t-1} + \frac{r^F b^F}{z}\hat{r}^F_{t-1} - \frac{r^F b^F}{z}\hat{z}_t \tag{80}$$

$$0 = -\hat{r}^F_t + \hat{r}^*_t - \frac{\psi_b}{y}\tilde{b}^F_t + \frac{\psi_b b^F}{y}\hat{y}_t + \zeta^b_t \tag{81}$$

$$0 = g\hat{g}_t + \frac{b}{\beta}\hat{r}_{t-1} + \frac{1}{\beta}\tilde{b}_{t-1} - \frac{b}{\beta}\hat{z}_t - \tilde{b}_t - c\tilde{\tau}^c_t - \tau^c c\hat{c}_t - wl\tilde{\tau}^w_t - wl\tau^w(\hat{w}_t + \hat{l}_t) - \frac{r^K k}{z}\tilde{\tau}^K_t - \frac{\tau^K r^K k}{z}(\hat{r}^K_t + \hat{k}_{t-1} - \hat{z}_t) - \tilde{\tau}_t \tag{82}$$

$$0 = -\hat{g}_t + \rho_g\hat{g}_{t-1} - (1-\rho_g)\gamma_{gb}\left(\frac{1}{y}\tilde{b}_{t-1} - \frac{b}{y}\hat{y}_{t-1}\right) + \varepsilon_{g,t} \tag{83}$$

$$0 = -\tilde{\tau}^c_t + \rho_c\tilde{\tau}^c_{t-1} + (1-\rho_c)\gamma_{cb}\left(\frac{1}{y}\tilde{b}_{t-1} - \frac{b}{y}\hat{y}_{t-1}\right) + \varepsilon_{c,t} \tag{84}$$

$$0 = -\tilde{\tau}^w_t + \rho_w\tilde{\tau}^w_{t-1} + (1-\rho_w)\gamma_{wb}\left(\frac{1}{y}\tilde{b}_{t-1} - \frac{b}{y}\hat{y}_{t-1}\right) + \varepsilon_{w,t} \tag{85}$$

$$0 = -\tilde{\tau}_t^k + \rho_k \tilde{\tau}_{t-1}^k + (1-\rho_K)\gamma_{Kb}\left(\frac{1}{y}\tilde{b}_{t-1} - \frac{b}{y}\hat{y}_{t-1}\right) + \varepsilon_{k,t} \tag{86}$$

$$0 = -\tilde{\tau}_t + \rho_\tau \tilde{\tau}_{t-1} + (1-\rho_\tau)\gamma_{\tau b}\left(\frac{1}{y}\tilde{b}_{t-1} - \frac{b}{y}\hat{y}_{t-1}\right) + \varepsilon_{\tau,t} \tag{87}$$

$$0 = \hat{z}_t - \rho_z \hat{z}_{t-1} - \varepsilon_{z,t} \tag{88}$$

$$0 = \hat{r}_t^* - \rho_{r^*}\hat{r}_{t-1}^* - \varepsilon_{r^*,t} \tag{89}$$

$$0 = \hat{\zeta}_t - \rho_\zeta \hat{\zeta}_{t-1} - \varepsilon_{\zeta,t} \tag{90}$$

$$0 = \hat{\zeta}_t^L - \rho_L \hat{\zeta}_{t-1}^L - \varepsilon_{L,t} \tag{91}$$

$$0 = \hat{\zeta}_t^I - \rho_I \hat{\zeta}_{t-1}^I - \varepsilon_{I,t} \tag{92}$$

$$0 = \zeta_t^b - (1-\rho_b)\zeta^b - \rho_b \zeta_{t-1}^b - \varepsilon_{b,t} \tag{93}$$

1.4.2 Observation Equations

$$\Delta\hat{y}_t^{obs} = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{94}$$

$$\Delta\hat{c}_t^{obs} = \hat{c}_t - \hat{c}_{t-1} + \hat{z}_t \tag{95}$$

$$\hat{\frac{g}{y}}_t^{obs} = \hat{g}_t - \hat{y}_t \tag{96}$$

$$\tilde{nx}_t^{obs} = \frac{1}{y}\tilde{nx}_t - \frac{nx}{y}\hat{y} \tag{97}$$

$$\Delta\hat{w}_t^{obs} = \hat{w}_t - \hat{w}_{t-1} + \hat{z}_t \tag{98}$$

$$\hat{r}_t^{obs} = \hat{r}_t \tag{99}$$

$$\hat{r^*}_t^{obs} = \hat{r^*}_t \tag{100}$$

$$\left(\frac{\tilde{b}}{y}\right)_t^{obs} = \frac{1}{y}\tilde{b}_t - \frac{b}{y}\hat{y}_t \tag{101}$$

$$\left(\frac{\hat{\tau^c c}}{y}\right)_t^{obs} = \frac{c}{y}\tilde{\tau}_t^c + \frac{\tau^c c}{y}\hat{c}_t - \frac{\tau^c c}{y}\hat{y}_t \tag{102}$$

$$\left(\frac{\hat{\tau^w wl}}{y}\right)_t^{obs} = \frac{wl}{y}\tilde{\tau}_t^w + \frac{\tau^w wl}{y}\hat{w}_t + \frac{\tau^w wl}{y}\hat{l}_t - \frac{\tau^w wl}{y}\hat{y}_t \tag{103}$$

$$\left(\frac{\hat{\tau^K r^K k}}{y}\right)_t^{obs} = \frac{r^K k}{y}\tilde{\tau}_t^K + \frac{\tau^K r^K k}{y}\hat{r}_t^K + \frac{\tau^K r^K k}{y}\hat{k}_{t-1} - \frac{\tau^K r^K k}{y}\hat{y}_t \tag{104}$$

2 Data Sources

This section describes the data used to estimate the model.

- **Population:** Quarterly gross domestic product in chain volume measure (ABS Catalogue 5206.001) divided by quarterly gross domestic product per capita also in chain volume measure (ABS Catalogue 5206.001).

- **Real GDP per capita:** Quarterly gross domestic product per capita in chain volume measure (ABS Catalogue 5206.001). This series enters in first difference in the estimation.

- **Consumption per capita:** Quarterly private consumption in chain volume measure (ABS Catalogue 5206.002) divided by population. This series enters in first difference in the estimation with its sample mean adjusted to match the sample mean of real output growth.

- **Government spending-to-GDP ratio:** Quarterly government consumption and pubic gross fixed capital formation in current prices (ABS Catalogue 5206.003) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003). This series enters in log form in the estimation.

- **Net exports-to-GDP ratio:** Net exports-to-GDP is computed as exports-to-GDP less imports-to-GDP. Exports-to-GDP is quarterly exports in current price measure divided by quarterly gross domestic product in current prices. Imports to-GDP is quarterly imports in current prices divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003). The sample mean of this series is removed prior to the estimation.

- **Hourly wage:** Compensation of employees (ABS Cat 5206.044) divided by the hours worked index (ABS Cat 5206.001). The series is deflated by the consumption deflator (ABS Cat 5206.005). This series enters in first difference with its sample mean adjusted to equal the mean of output growth.

- **Domestic Real interest rate:** 90-day bank bill rate (RBA Bulletin Table F1). This nominal interest rate is converted to a real rate using the trimmed mean inflation series (RBA Bulletin Table G1). The monthly series is converted into quarterly frequency by arithmetic averaging.

- **Foreign Real interest rate:** 3-months U.S. Treasury bill rate (FRED Database). This nominal interest rate is converted to a real rate using the U.S. core PCE inflation series (FRED Database). The monthly series is converted into quarterly frequency by arithmetic averaging.

- **Government debt-to-GDP-ratio:** Commonwealth government securities on issue (Australian Office of Financial Management and RBA Bulletin Table E3) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003).

- **Consumption tax revenues-to-GDP-ratio:** The sum of sales tax revenues and goods and services tax revenues in current prices (ABS Cat 5206.022) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003). The mean of the series is adjusted for the subsample 1983-2000 to adjust for the break resulting from the introduction of the goods and services tax in the year 2000.

- **Labour income tax revenues-to-GDP ratio:** Individual income tax revenues in current prices (ABS Cat 5206.022) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003).

- **Capital income tax revenues-to-GDP ratio:** The sum of resident corporations' income tax revenues and non-residents' income tax revenues in current prices (ABS Cat 5206.022) divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003)

Modelling Structural Break in Trend Growth

The model is estimated and solved using the technique developed in Kulish and Pagan (2017) for models with structural changes. We allow for the structural break in steady-state trend growth, z, and in the variance of shocks to happen at possibly different dates in the sample, T_z and T_σ. Hence, for the data sample $t = 1, 2, \cdots, T$, and assuming that $T_z < T_\sigma$, three different regimes occur:

1. First regime: For $t = 1, 2, \cdots, T_z - 1$, steady-state labour-augmenting technology growth takes an initial value, z. In the initial regime, the first-order approximation to the equilibrium conditions around the steady state is a linear rational expectations system of equations that is given by:

$$A_0 y_t = C_0 + A_1 y_{t-1} + \mathbb{E}_t B_0 y_{t+1} + D_0 \varepsilon_t \tag{105}$$

   where the structural matrices A_0, C_0, A_1, B_0 and D_0 correspond to the initial steady state, y_t is vector of state and jump variables and ε_t is a vector of exogenous *iid* shocks. The solution, if it exists and is unique, will be a Vector Autoregression (VAR) that takes the form:

$$y_t = C + Q y_{t-1} + G \varepsilon_t \tag{106}$$

2. Second regime: For $t = T_z, \cdots, T_\sigma - 1$, steady-state labour-augmenting technology growth takes a different value, say z'. The structural form of the model then becomes:

$$A_0^* y_t = C_0^* + A_1^* y_{t-1} + \mathbb{E}_t B_0^* y_{t+1} + D_0 \varepsilon_t \tag{107}$$

   where the superscript $*$ is associated with the matrices that correspond to the new steady-state commodity price level. Note that the matrix D_0 is unchanged as the break in the variances of shocks hasn't occurred yet. The solution, if it exists and is unique, will be a VAR that takes the form:

$$y_t = C^* + Q^* y_{t-1} + G^* \varepsilon_t \tag{108}$$

3. Third regime: For $t = T_\sigma, \cdots, T$, the variances of shocks change. The structural form of the model then becomes:

$$A_0^* y_t = C_0^* + A_1^* y_{t-1} + \mathbb{E}_t B_0^* y_{t+1} + D_0^{**} \varepsilon_t \tag{109}$$

where the matrix D_0^{**} denotes the matrix corresponding to the new variances of shocks while other structural matrices are maintained as in the second regime. The solution, if it exists and is unique, will be a VAR that takes the form:

$$y_t = C^* + Q^* y_{t-1} + G^{**} \varepsilon_t \tag{110}$$

Based on the three regimes, the time-varying reduced form is given by:

$$y_t = C_t + Q_t y_{t-1} + G_t \varepsilon_t \tag{111}$$

Given a data sample, one can form an observable variables vector, y_t^{obs}, that relates to the variables in the model by:

$$y_t^{obs} = H y_t + v_t \tag{112}$$

where v_t is a vector of *iid* measurement errors with zero mean and covariance matrix V. Together, the state equation, Equation (111), and the observation equation, Equation (112), form a state-space model. Hence, the data sample's likelihood function can be constructed by using the Kalman filter as outlined in Kulish and Pagan (2017).

3 Growth Accounting Calculations for Australia

To perform the growth accounting exercise, we assume Australia's output per capita can be modelled as a Cobb-Douglas aggregate of available technology and capital per capita:

$$y_t = A_t k_t^{\alpha} \tag{113}$$

where y_t is output per capita, A_t is total factor productivity, and k_t is capital per capita. Hence, output per capita growth, g_y, is given as:

$$g_y = g_a + \alpha g_k \tag{114}$$

where g_a is the contribution of total factor productivity to output per capita growth and αg_k is the contribution of capital per capita of output growth. The results of the growth accounting calculations for Australia are given in Table 1.

Table 1: Growth Accounting Calculations for Australia

Period	Average GDP per capita growth %	Contribution of capital per capita %	Contribution of total factor productivity %
1990-2000	2.02	0.61	1.41
1990-2017	1.65	0.70	0.95
2000-2017	1.36	0.77	0.59
2010-2017	1.10	0.70	0.40

Below is a description of the data used in the growth accounting calculation:

- **Population:** Annual gross domestic product in chain volume measure (ABS Catalogue 5204.0) divided by annual gross domestic product per capita also in chain volume measure (ABS Catalogue 5204.0).

- **Real GDP per capita:** Gross domestic product using the production based approach in chain volume measure (ABS Catalogue 5204.0) divided by population.

- **Capital per capita:** End-year net capital stock in chain volume measure (ABS catalogue 5204.0) divided by population.

- **Capital share in production function:** The ratio of gross operating surplus in all sectors to income. Income is computed as the sum of compensation of employees (ABS Catalogue 5204.0) and gross operating surplus in all sectors (ABS Catalogue 5204.0).

4 Unobserved Components Estimates

We set up linear and non-linear unobserved components trend-cycle decomposition models for the quarterly level of GDP and allow for a break in output trend to happen at any date as well as a break in the variance of the shock to the trend and variance of the shock to the cycle to occur on the same date.

4.1 Linear Unobserved Components Model

The linear unobserved components trend-cycle decomposition model is given by:

$$y_t = \tau_t + c_t \tag{115}$$

$$\tau_t = z\mathbf{1}(t < T_z) + (z + \Delta z)\mathbf{1}(t \geq T_z) + \tau_{t-1} + \epsilon_t^\tau \tag{116}$$

$$c_t = \rho_1 c_{t-1} + \rho_2 c_{t-2} + \epsilon_t^c \tag{117}$$

where y_t is the logarithm of Australia's real GDP per capita which is decomposed into a trend component τ_t and a cyclical component c_t. The trend component τ_t is specified as a random walk with a drift and we allow for a break in the drift to happen at the date T_z. $\mathbf{1}(A)$ is an indicator function that takes the value 1 if the condition A is true and a value of 0 otherwise. As such, the mean growth rate of the trend equals z before the break date T_z, and $z' = z + \Delta z$ on and after the break date. The cyclical component c_t is modelled as a zero-mean stationary AR(2) process. We assume that the innovations ϵ_t^τ and ϵ_t^c are independently normal:

$$\begin{pmatrix} \epsilon_t^\tau \\ \epsilon_t^c \end{pmatrix} = \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} \mu\sigma_\tau^2\mathbf{1}(t < T_\sigma) + \sigma_\tau^2\mathbf{1}(t \geq T_\sigma) & 0 \\ 0 & \mu\sigma_c^2\mathbf{1}(t < T_\sigma) + \sigma_c^2\mathbf{1}(t \geq T_\sigma) \end{bmatrix}\right)$$

We allow for a break in the variances of the innovations ϵ_t^τ and ϵ_t^c to occur at the same date T_σ. As such, the variances of the shocks to the trend and the cycle are respectively $\mu\sigma_\tau^2$ and $\mu\sigma_c^2$ before the break date T_σ, and σ_τ^2 and σ_c^2 on and after the break date.

The linear unobserved components trend-cycle decomposition model can be written is state space form:

$$y_t = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x_t \tag{118}$$

$$x_t = \begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} \mathbf{1}(t < T_z) + \begin{bmatrix} z' \\ 0 \\ 0 \end{bmatrix} \mathbf{1}(t \geq T_z) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_1 & \rho_2 \\ 0 & 1 & 0 \end{bmatrix} x_{t-1} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_t^\tau \\ \epsilon_t^c \end{bmatrix} \tag{119}$$

where $x_t = \begin{bmatrix} \tau_t & c_t & c_{t-1} \end{bmatrix}'$.

To estimate the model, we calibrate the growth rate in the initial regime at 0.0055 as in the small open economy model and use a Bayesian estimation technique to estimate the remaining parameters (ϑ) and the break dates ($\mathbf{T}$). We set the priors to either be in consistence with the literature or to be uninformative. Uniform prior with support -0.0045 to 0.015 is set for the mean growth of the trend parameter z' . Normal distribution with mean 0.9 and standard deviation 1 is imposed on the autoregressive parameter ρ_1. For the autoregressive parameter ρ_2, we impose a normal prior with mean 0 and standard deviation 1. The priors on the standard deviations of shocks, σ_τ and σ_c are set as uniform priors with support 0 and 0.2. Further, a uniform prior $[0, 3]$ is imposed on the variance scale parameter μ. Finally, flat priors are imposed for the break date T_z and T_σ and the initial regime is restricted to be at least 60 quarters long. The prior and posterior distributions of the parameters from estimating the model at level and at first-difference are listed in Tables 2 and 3, respectively.

Table 2: Prior and Posterior Distribution of the Parameters and Break Dates from Level Estimation

	Prior distribution			Posterior distribution			
Parameter	Dist.	Mean	S.d.	Mean	Mode	5%	95%
Parameters							
z'	Uniform	$[-0.0045, 0.015]$		0.0025	0.0029	0.0014	0.0036
ρ_1	Normal	0.9	1	0.8344	0.8907	0.1365	1.4520
ρ_2	Normal	0	1	-0.2197	0.0017	-0.7982	0.4072
σ_τ	Uniform	$[0, 0.2]$		0.0074	0.0079	0.0031	0.0094
σ_c	Uniform	$[0, 0.2]$		0.0026	0.0004	0.0002	0.0076
μ	Uniform	$[0, 3]$		1.9941	1.8537	1.5998	2.4486
T_z	Flat	[1997:Q4, 2015:Q2]		2006:Q3	2008:Q1	2002:Q2	2008:Q4
T_σ	Flat	[1997:Q4, 2015:Q2]		2002:Q1	2004:Q2	1998:Q2	2005:Q2

Table 3: Prior and Posterior Distribution of the Parameters and Break Dates from First-Difference Estimation

	Prior distribution			Posterior distribution			
Parameter	Dist.	Mean	S.d.	Mean	Mode	5%	95%
Parameters							
z'	Uniform	$[-0.0045, 0.015]$		0.0025	0.0030	0.0014	0.0036
ρ_1	Normal	0.9	1	0.8237	0.9620	0.1286	1.4446
ρ_2	Normal	0	1	-0.2154	0.0000	-0.8041	0.4214
σ_τ	Uniform	$[0, 0.2]$		0.0074	0.0001	0.0029	0.0096
σ_c	Uniform	$[0, 0.2]$		0.0026	0.0078	0.0002	0.0076
μ	Uniform	$[0, 3]$		2.0051	1.8420	1.6041	2.4713
T_z	Flat	[1997:Q4, 2015:Q2]		2006:Q3	2008:Q1	2002:Q2	2008:Q4
T_σ	Flat	[1997:Q4, 2015:Q2]		2002:Q1	2004:Q2	1998:Q2	2005:Q2

4.2 Non-linear Unobserved Components Model

The non-linear unobserved components model is set up as a Friedman's Plucking model as in Kim and Nelson (1999). Here, the trend-cycle decomposition is given by:

$$y_t = \tau_t + c_t \tag{120}$$

$$\tau_t = z\mathbf{1}(t < T_z) + (z + \Delta z)\mathbf{1}(t \geq T_z) + \tau_{t-1} + \epsilon_t^\tau \tag{121}$$

$$c_t = \rho_1 c_{t-1} + \rho_2 c_{t-2} + \pi_{S_t} + \epsilon_t^c \tag{122}$$

$$\pi_{S_t} = \pi S_t, \qquad \pi \neq 0 \tag{123}$$

where π_{S_t} is an asymmetric, discrete, shock which is dependent upon an unobserved variable S_t. We assume that S_t evolves according to a first-order Markov-switching process as in Hamilton (1989):

$$Pr[S_t = 1 | S_{t-1} = 1] = p \tag{124}$$

$$Pr[S_t = 0 | S_{t-1} = 0] = q \tag{125}$$

As in the linear model, the trend component τ_t is specified as a random walk with a drift and we allow for a break in the drift to happen at the date T_z.

The non-linear unobserved components trend-cycle decomposition model can be written is state space form:

$$y_t = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x_t \tag{126}$$

$$x_t = \begin{bmatrix} z \\ \pi_{S_t} \\ 0 \end{bmatrix} \mathbf{1}(t < T_z) + \begin{bmatrix} z' \\ \pi_{S_t} \\ 0 \end{bmatrix} \mathbf{1}(t \geq T_z) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_1 & \rho_2 \\ 0 & 1 & 0 \end{bmatrix} x_{t-1} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_t^{\tau} \\ \epsilon_t^{c} \end{bmatrix} \tag{127}$$

where $x_t = \begin{bmatrix} \tau_t & c_t & c_{t-1} \end{bmatrix}'$.

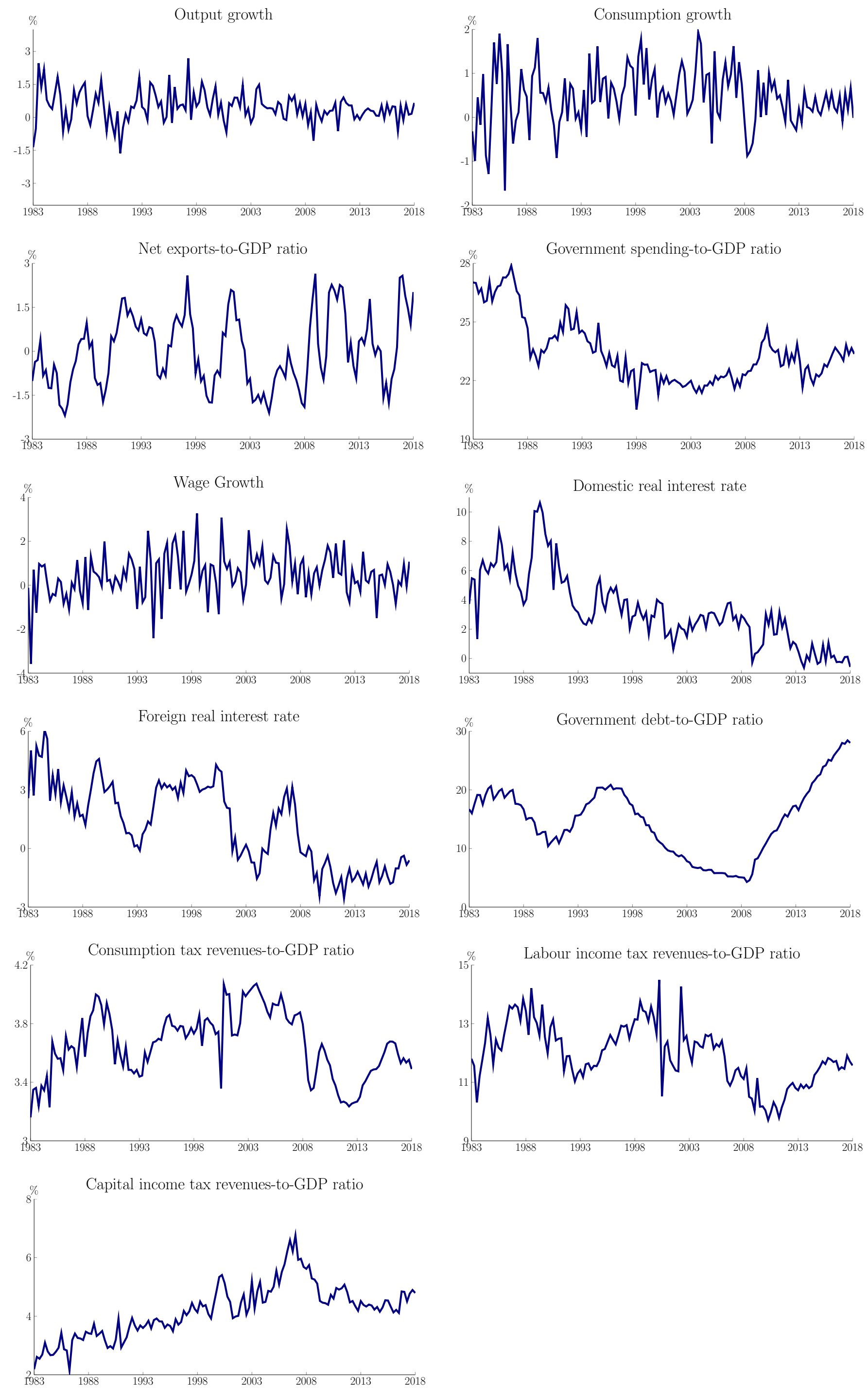
To estimate the non-linear model, we calibrate the growth rate in the initial regime at 0.0055 as in the small open economy model and use a Bayesian estimation technique to estimate the remaining parameters (ϑ) and the break dates ($\mathbf{T}$). In estimation, the Kim (1994) filter is used which combines the Kalman filter with Hamilton (1989) filter for Markov-switching models. The prior and posterior distributions of the parameters from estimating the model at level are listed in Table 4.

Table 4: Prior and Posterior Distribution of the Parameters and Break Dates from Level Estimation

	Prior distribution			Posterior distribution			
Parameter	Dist.	Mean	S.d.	Mean	Mode	5%	95%
Parameters							
z'	Uniform	$[-0.0045, 0.015]$		0.0025	0.0029	0.0015	0.0035
ρ_1	Normal	0.9	1	1.0420	0.5306	0.2979	1.5729
ρ_2	Normal	0	1	-0.3338	0.0004	-0.7660	0.1852
σ_τ	Uniform	$[0, 0.2]$		0.0072	0.0079	0.0057	0.0092
σ_c	Uniform	$[0, 0.2]$		0.0004	0.0028	0.0003	0.0061
μ	Uniform	$[0, 3]$		2.0993	1.9003	1.6438	2.6442
π	Uniform	$[-0.05, 0.05]$		0.0015	-0.0015	-0.0041	0.0063
p	Beta	0.05	0.15	0.0504	0.0000	0.0026	0.1446
q	Beta	0.25	0.1	0.2605	0.2241	0.1196	0.4275
T_z	Flat	[1997:Q4, 2015:Q2]		2006:Q2	2007:Q2	2001:Q3	2008:Q3
T_σ	Flat	[1997:Q4, 2015:Q2]		2002:Q1	2004:Q1	1998:Q1	2008:Q1

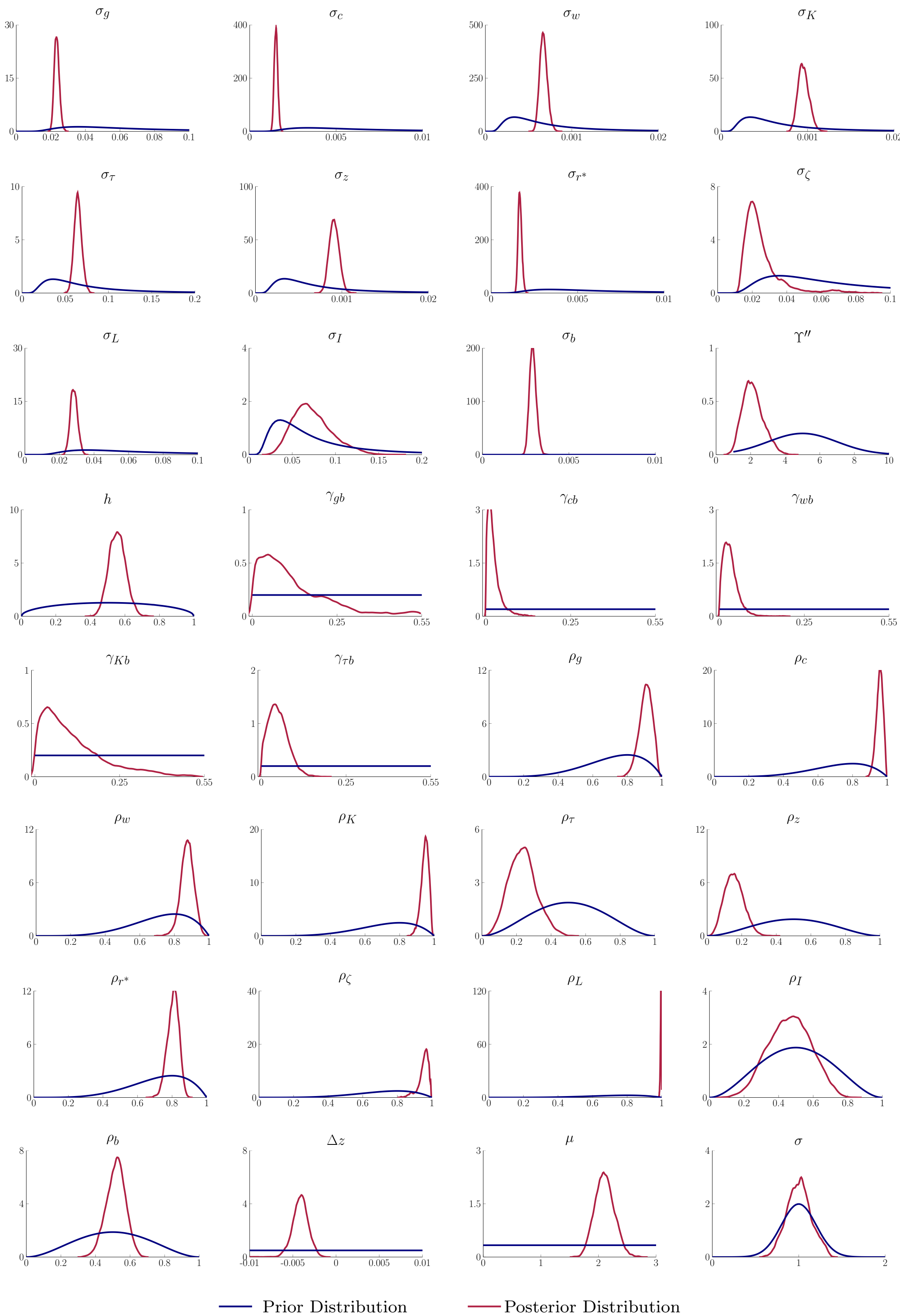
5 Additional Figures

Figure 1: Observable Variables Used in Estimation



Sources: ABS; AOFM; Authors' calculations; FRED; RBA

Figure 2: Prior and Posterior Distributions



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